3, 4, 2, 4 13 total

MATH 135, Problem Set 5 B solutions

3.2/66

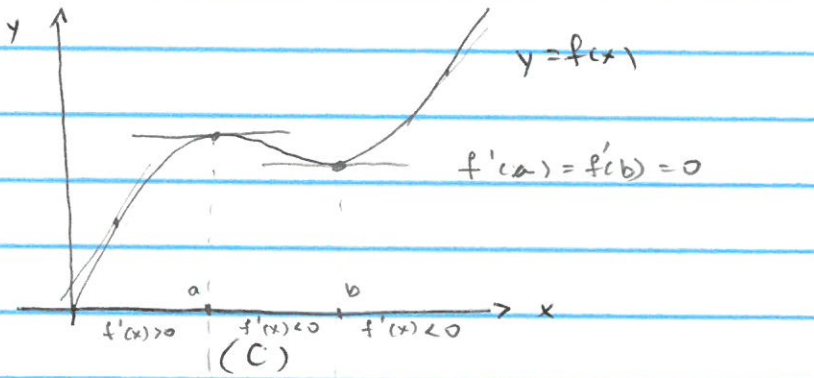
If $f(x)$ is in graph (A), $f'(x)$ is (III)

(3) (B) (I)

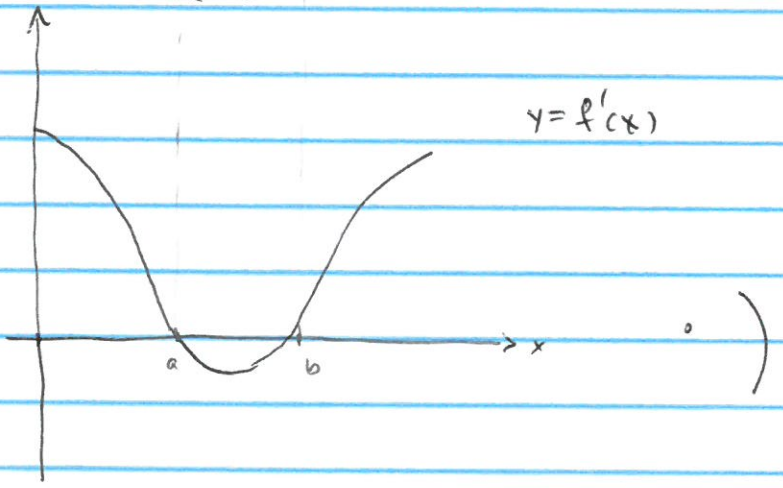
(C) (II)

(Say "see solution" if they miss any of these)

(For example



(C)



92. At a general $x = a$, the tangent line to $y = 1 - x^2$ is $y - (1-a^2) = (-2a)(x-a)$, or

$$y = -2ax + (a^2+1) \quad (1)$$

the intersections of this line and the other parabola

are found by setting

$$-2ax + (a^2+1) = 4 - (x-4)^2$$

or $$x^2 - 8x + 12 - 2ax + a^2 + 1 = 0$$

or $$x^2 - (2a+8)x + 12 + a^2 + 1 = 0$$ (1)

This has just one real root when the discriminant satisfies

$$(2a+8)^2 - 4(12 + a^2 + 1) = 0$$

$$4a^2 + 32a + 64 - 4a^2 - 52 = 0$$

$$32a = -12$$

$$\boxed{a = -\frac{12}{32} = -\frac{3}{8}}$$ (1)

The slope of the board is $\underbrace{(-2)\left(-\frac{3}{8}\right)}_{\text{from the derivative of } 1-x^2} = \boxed{\frac{3}{4}}$ (1)

(If they don't do this, say "read the problem again" and give answer)

3.3/38

$$\frac{d}{dx}\left(\frac{ax+b}{cx+d}\right) = \frac{(cx+d)\cdot a - (ax+b)\cdot c}{(cx+d)^2}$$ quotient rule

(OK if unsimplified)

$$= \boxed{\frac{ad-bc}{(cx+d)^2}}$$ (2)

45. (a) $\frac{d}{dx}(e^{2x}) = \frac{d}{dx}(e^x \cdot e^x) = e^x \cdot e^x + e^x \cdot e^x = \boxed{2e^{2x}}$ (1)

(b) $\frac{d}{dx}(e^{4x}) = \frac{d}{dx}(e^{2x} \cdot e^{2x}) = e^{2x} \cdot 2e^{2x} + 2e^{2x} \cdot e^{2x}$ by (a)

$= \boxed{4e^{4x}}$. ①

(c) $\frac{d}{dx}(e^{8x}) = \frac{d}{dx}(e^{4x} \cdot e^{4x})$

$= e^{4x} \cdot 4e^{4x} + e^{4x} \cdot 4e^{4x}$ log (b)

$= \boxed{8e^{8x}}$ ①

(d) Looks like $\boxed{\frac{d}{dx}(e^{ax}) = ae^{ax}}$! ①