4, 2, 4, 4   14 total

MATH 135 - Problem Set 4, B Solutions

§ 2.7 / 32. using the given hint:

$$\lim_{x\to\infty} \sqrt{x^2+1} - x = \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}+x}$$

$$= 0$$ 1 (since by Theorem 1 on p. 95, the numerator is constant and the denominator $\to +\infty$ as $x \to +\infty$)

[Note, the Hint is true because

$$\sqrt{x^2+1} - x = \left(\sqrt{x^2+1} - x\right)\cdot\frac{\left(\sqrt{x^2+1}+x\right)}{\left(\sqrt{x^2+1}+x\right)}$$

$$= \frac{x^2+1-x^2}{\sqrt{x^2+1}+x}$$ 2

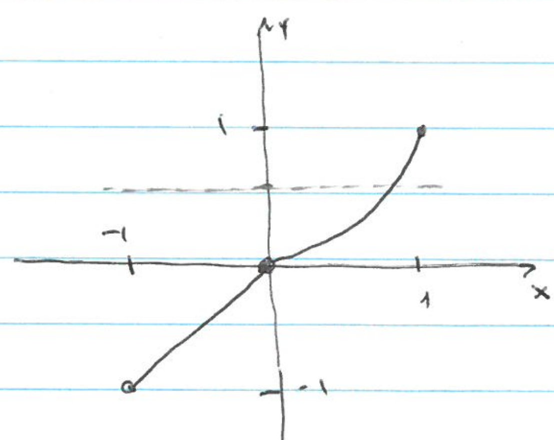
$$= \frac{1}{\sqrt{x^2+1}+x}.\ ]$$

§ 2.8 / 18. $f(x) = \begin{cases} x & \text{for } x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases}$ is continuous at 0 1

$-\frac{1}{2}$ for omitting this

since $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x = 0 = \underbrace{\lim_{x\to 0^+} x^2}$ $= \lim_{x\to 0^+} f(x)$ and

$f(0) = 0$. It is also continuous at all other x in $[-1, 1]$:

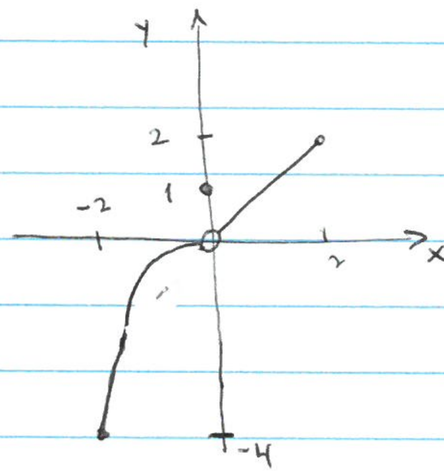


Hence the IVT does apply 1 show $f(x)$ takes on all values between $f(-1) = -1$ and $f(1) = 1$.

20. $f(x) = \begin{cases} -x^2 & \text{for } x < 0 \\ 1 & \text{for } x = 0 \\ x & \text{for } x > 0 \end{cases}$ on $[-2, 2]$

Note: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} -x^2 = 0 \neq 1 = f(0)$. Hence

f is not continuous at $x = 0$ in $[-2, 2]$. 1 This means the IVT does not apply. 1



2

Moreover, it is clear from a graph like this that, even though 0 is between $f(-2) = -4$ and $f(2) = 2$, there is no x in $[-2, 2]$ with $f(x) = 0$.

§ 3.1/48 Using the grid in Figure 15 (B), we estimate the slope of the tangent is about

$$m = \frac{4.01 - 4}{16.1 - 16} \doteq \frac{.01}{.1} \doteq \boxed{.1}$$

2

Anything close to this is OK

(There are 9 grid lines between 4 and 4.1 on the y-axis, so the y-coordinate at $x = 16.1$ is about 4.01, since it's $\doteq .1$ of the way between 4 and 4.1.)

So we estimate $f'(16) \doteq .1$. The exact value is

$$f'(16) = \lim_{h \to 0} \frac{\sqrt{16+h} - \sqrt{16}}{h}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{16+h} - \sqrt{16}\right)}{h} \cdot \frac{\left(\sqrt{16+h} + \sqrt{16}\right)}{\left(\sqrt{16+h} + \sqrt{16}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{16} + h - \cancel{16}}{h\left(\sqrt{16+h} + \sqrt{16}\right.}$$ now cancel h's

$$= \lim_{h \to 0} \frac{1}{\sqrt{16+h} + \sqrt{16}}$$

$$= \frac{1}{8} \doteq .125$$ (reasonably close to estimate)

2

(It's <u>OK</u> if they use $\frac{d}{dx} x^{1/2} = \frac{1}{2x^{1/2}}$ here)