points: 4, 6, 4, 2 = 16 total

MATH 135 - 01 Problem Set 2, Part B Solutions

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We proceed as usual to give the formula for the inverse function. We solve $y = \ln(x^2 - 2)$ = f(x) for x:

part credit: 1 for exponential
1 for square root
1 for choice of sign
1 for final formula

$$y = \ln(x^2 - 2)$$
$$e^y = x^2 - 2 \quad 1$$
$$\pm\sqrt{e^y + 2} = x \quad 1$$

To get $x > \sqrt{2}$, 1 we need to take the + sign, and then $\boxed{f^{-1}(x) = \sqrt{e^x + 2}.}$ (Can also see this as follows: the domain of $f(x) = \ln(x^2 - 2)$ = $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$. This function is only 1-1 if the domain is restricted to $(\sqrt{2}, \infty)$ or $(-\infty, -\sqrt{2})$. If we choose the first interval $(\sqrt{2}, \infty)$, then the range of f^{-1} must be $(\sqrt{2}, \infty)$, and that only happens with the + sign.)

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anything within .05 of these is OK

(a) line A passes through (4, .19) and (6, .28) (roughly). The slope represents the average rate of change of the fraction infected between 4 and 6 weeks 1

Estimate of slope: $\dfrac{.28 - .19}{6 - 4} \doteq .045$ 1 Anything close is OK

Line B is tangent to the graph at (6, .28) (roughly). The slope represents the instantaneous rate of change of the fraction infected at $t = 6$ weeks. 1

Estimate of slope (using the intercept at (0, .15) roughly

$\dfrac{.28 - .15}{6 - 0} \doteq .0217$ 1 Again close is OK

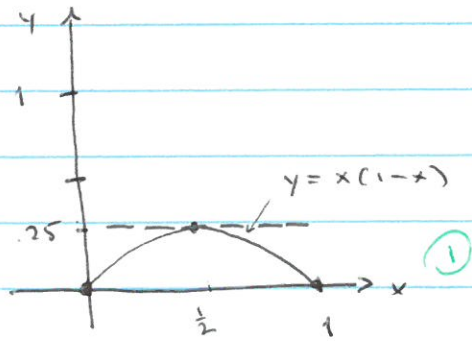
(b) It seems the slopes of the tangent lines at $t = 1, 2, 3$ are increasing as t increases. So at $\boxed{t=3}$. 1

No part credit, but give correct value if they miss this

(c) The slopes of the tangents seem to be decreasing as t increases in this part of the graph. So at $\boxed{t=4}$ 1

same

30.



(a downward-opening parabola). 1

(a) (Average rate of change over $[0,1]$) $= 0$, since $f(x) = x(1-x)$ satisfies $f(0) = f(1) = 0$. Hence $\frac{f(1)-f(0)}{1-0} = \frac{0-0}{1} = 0$. 1

(b) The tangent line to the parabola is horizontal at $t = \frac{1}{2}$, so instantaneous rate of change is $\underline{\underline{0}}$. 1

(c) rate of change is positive for $0 \le x < \frac{1}{2}$ 1
(i.e. slope of tangent)

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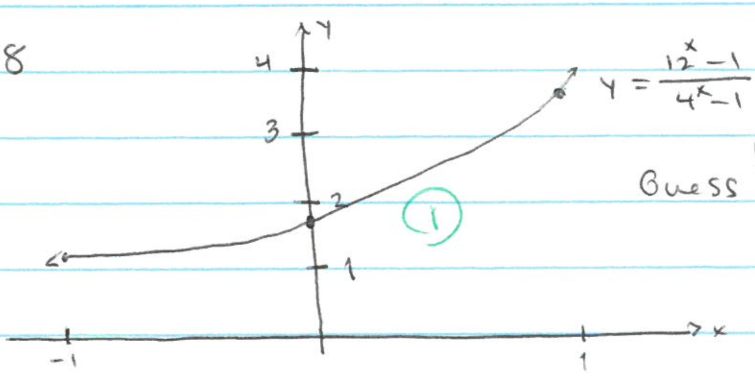

1

Guess $\boxed{\lim_{x \to 0} \frac{12^x - 1}{4^x - 1} \doteq 1.8}$ 1

(using graph)

Note: exact value is $\frac{\ln(12)}{\ln(4)} \doteq 1.792$
but the estimate 1.8 or anything close is OK