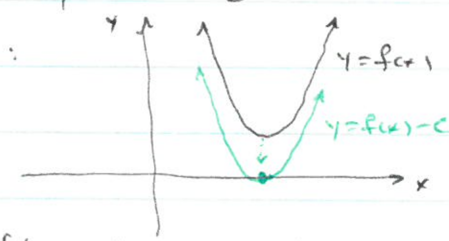
Points: 4, 8, 2, 4, 4   Total: 22

MATH 135 – Problem Set 1, Part B Solutions

1.2/

48. (a) is correct. (b) is not. (1) If $f(x) = Ax^2 + Bx + C$ is a quadratic function ($A \neq 0$), then the graph $y = f(x)$ is a parabola (1) opening up if $A > 0$, down if $A < 0$. $y = f(x) - c$ will give the same parabola shifted up or down depending on the sign of c. There will be exactly one c where $y = f(x) - c$ is tangent to the x-axis: (2)



For that c, $f(x) - c = 0$ has a double root. $y = f(x-c)$ shifts the graph left or right depending on the sign of c. That cannot produce a double root if $y = f(x) = 0$ has two distinct roots, or no real roots.

54. (1) We have for $P = (x, y)$, $d_1 = \sqrt{(x-0)^2 + (y-\frac{1}{4})^2}$ (1) and $d_2 = y + \frac{1}{4}$ (1). If $d_1 = d_2$ then

$$\sqrt{x^2 + (y-\tfrac{1}{4})^2} = (y + \tfrac{1}{4})$$

squaring: $$x^2 + (y - \tfrac{1}{4})^2 = (y + \tfrac{1}{4})^2,$$

so $$x^2 + \cancel{y^2} - \tfrac{1}{2}y + \cancel{\tfrac{1}{16}} = \cancel{y^2} + \tfrac{1}{2}y + \cancel{\tfrac{1}{16}}$$ (2)

and $$x^2 = y.$$

Hence if $d_1 = d_2$, then P lies on the parabola $y = x^2$.

(2) If $y = x^2$, then $d_1 = \sqrt{x^2 + (x^2 - \frac{1}{4})^2}$ (1)

and $d_2 = x^2 + \frac{1}{4}$. (1) We claim $d_1 = d_2$. This follows by simplifying the formula for d_1:

$$\begin{aligned} d_1 &= \sqrt{x^2 + (x^2 - \tfrac{1}{4})^2} \\ &= \sqrt{x^2 + x^4 - \tfrac{1}{2}x^2 + \tfrac{1}{16}} \\ &= \sqrt{x^4 + \tfrac{1}{2}x^2 + \tfrac{1}{16}} \quad \text{after combining the } x^2 \text{ terms.} \\ &= \sqrt{(x^2 + \tfrac{1}{4})^2} \quad \text{after factoring.} \\ &= x^2 + \tfrac{1}{4} \\ &= d_2 . \end{aligned}$$

(2)

1.3/40 $\quad f(x) = \dfrac{x+1}{x^2 + 2cx + 4} = \dfrac{x+1}{(x+c)^2 + 4 - c^2}$

after completing the square in the denominator. this will have domain all $x \in \mathbb{R}$ as long as $4 - c^2 > 0$ or $c^2 < 4$ or $-2 < c < 2$. (2) the idea is that those are the c-values where $x^2 + 2cx + 4 \neq 0$ for all x. (Note: this can also be done with the Quadratic Formula: $x^2 + 2cx + 4 = 0$ when $x = \dfrac{-2c \pm \sqrt{4c^2 - 16}}{2} = -c \pm \sqrt{c^2 - 4}$. there are no real roots when $c^2 - 4 < 0$ or $-2 < c < 2$.)

1.4/40. $\quad f(x) = -2\cos\left(\frac{3x}{2}\right)$ (2) is one such function. The amplitude is $\underline{A = 2}$ (1) and the period is $\underline{T = \frac{4\pi}{3}}$ (1).

60. The dotted line has length $b\sin\theta$. Hence from the Pythagorean Theorem in the right-side triangle,

$$c^2 = (b\sin\theta)^2 + (a - b\cos\theta)^2 \quad 1$$

$$= b^2\sin^2\theta + a^2 - 2ab\cos\theta + b^2\cos^2\theta$$

$$= a^2 + b^2(\sin^2\theta + \cos^2\theta) - 2ab\cos\theta$$

$$= a^2 + b^2 - 2ab\cos\theta \quad 2$$

(by the basic trig identity $\cos^2\theta + \sin^2\theta = 1$ for all θ)

the equation $c^2 = a^2 + b^2 - 2ab\cos\theta$ is the Law of Cosines.