Mathematics 135 – Calculus 1
Precalculus Review Problems
August, 2019

The following problems will not be collected or graded. They are similar to questions on the first diagnostic quiz to be given in class on Friday, September 6.

1. Find all values of x that satisfy the given inequality or inequalities:
   a) $-4x \geq 20$ – *Answer:* all $x \leq -5$ (dividing by -4 to solve for x flips the inequality because $-4 < 0$)
   b) $x + 1 > 4$, or $x + 2 < -1$ – *Answer:* all $x > 3$ together with all $x < -3$ (could also be written as a union of intervals: $(-\infty, -3) \cup (3, +\infty)$)
   c) $x + 3 > 1$ and $x - 2 < 1$ – *Answer:* The first inequality says $x > -2$ and the second says $x < 3$, so this is all x between -2 and 3, or $-2 < x < 3$ (could also be written as the interval $(-2, 3)$.)

2.
   a) Rewrite using positive exponents only: $\frac{x^{-1/3}}{x^{1/2}}$ – *Answer:* $\frac{1}{x^{5/6}}$
   b) Simplify: $(x^2y^{-3})(x^{-5}y^3)$ – *Answer:* $x^{-3} = \frac{1}{x^3}$
   c) Simplify: $\left(\frac{x^3}{-27y^{-6}}\right)^{-2/3}$ – *Answer:* $\frac{9}{x^2y^4}$
   d) Simplify: $\left(\frac{x^{-3}}{y^{-2}}\right)^2 \left(\frac{y}{x}\right)^4$ – *Answer:* $\frac{y^8}{x^{10}}$

3. A salesperson's monthly commission is 15% on all sales over \$12000. If the goal is to make a commission of at least \$3000 per month, what monthly sales figure should he or she attain? – *Answer:* Call the monthly sales figure x. (Assuming no commission on the sales under 12000), we want $(.15)(x - 12000) \geq 3000$ so $x \geq 32000$.
4. Factor:
   a) $7a^4 - 42a^2b^2 + 49a^3b$ – *Answer:* $7a^2(a^2 - 6b^2 + 7ab)$
   b) $xe^{-2x} - x^3e^{-x}$ – *Answer:* $xe^{-x}(e^{-x} - x^2)$
   c) $6ac + 3bc - 4ad - 2bd$ – *Answer:* $(2a + b)(3c - 2d)$
   d) $3x^2 - 6x - 24$ – *Answer:* $(3x + 6)(x - 4)$
   e) $9x^2 - 16y^4$ – *Answer:* $(3x - 4y^2)(3x + 4y^2)$ (difference of squares)

5. Solve for x:
   a. $x^2 + x - 12 = 0$ – *Answer:* $x = 3, -4$ (factor the quadratic as $(x - 3)(x + 4)$, or use the quadratic formula.)
   b. $4x^3 - 2x^2 - 2x = 0$ – *Answer:* $x = 0, 1, -1/2$ (factor as $2x(2x^2 - x - 1) = 2x(2x + 1)(x - 1)$
   c. $8x^2 - 8x - 3 = 0$ – *Answer:* $x = \frac{8 \pm \sqrt{64+96}}{16} = \frac{2 \pm \sqrt{10}}{4}$ (using the quadratic formula)

6. Simplify:

a. $\frac{2a^2-2b^2}{b-a} \cdot \frac{4a+4b}{a^2+2ab+b^2}$ – *Answer:* -8 (factor and cancel!)

b. $\frac{58}{3(3+t^2)} + \frac{1}{3}$ – *Answer:* $\frac{t^2+61}{3(3+t^2)}$

c. $\frac{2x}{2x-1} - \frac{3x}{2x+5}$ – *Answer:* $\frac{-2x^2+13x}{(2x-1)(2x+5)}$

d. $\frac{1+\frac{1}{x}}{1-\frac{1}{x^2}}$ – *Answer:* $\frac{x}{x-1}$

e. $\frac{2x(x+1)^{-1/2}-(x+1)^{1/2}}{x^2}$ – *Answer:* $\frac{x-1}{x^2\sqrt{x+1}}$

7. Let $f(x) = (x+1)^3$ and $g(x) = x^2 - 1$.
   a) What is the function $f(g(x))$? – *Answer:* $f(g(x)) = x^6$
   b) What is the function $g(f(x))$? – *Answer:* $g(f(x)) = (x+1)^6 - 1$
   c) What is the function $f(x)g(x)$? – *Answer:* $(x+1)^3(x^2-1) = (x+1)^4(x-1)$
   d) What is the domain of the function $\frac{f(x)}{g(x)}$? – *Answer:* all real numbers $x \neq -1, 1$.

8.
   a) Let $f(x) = x^3 - 2x^2 + 3$. Simplify as far as possible: $\frac{f(a+h)-f(a)}{h}$. – *Answer:*

$$\begin{aligned}\frac{f(a+h)-f(a)}{h} &= \frac{(a+h)^3 - 2(a+h)^2 + 3 - a^3 + 2a^2 - 3}{h} \\ &= \frac{a^3 + 3a^2h + 3ah^2 + h^3 - 2a^2 - 4ah - 2h^2 + 3 - a^3 + 2a^2 - 3}{h} \\ &= \frac{3a^2h + 3ah^2 + h^3 - 4ah - 2h^2}{h} \\ &= 3a^2 + 3ah + h^2 - 4a - 2h\end{aligned}$$

   b) Same question for $f(x) = \frac{1}{\sqrt{x}}$. – *Answer:*

$$\begin{aligned}\frac{\frac{1}{\sqrt{a+h}} - \frac{1}{\sqrt{a}}}{h} &= \frac{\sqrt{a} - \sqrt{a+h}}{h\sqrt{a}\sqrt{a+h}} \\ &= \frac{\sqrt{a} - \sqrt{a+h}}{h\sqrt{a}\sqrt{a+h}} \cdot \frac{\sqrt{a} + \sqrt{a+h}}{\sqrt{a} + \sqrt{a+h}} \\ &= \frac{-1}{\sqrt{a}\sqrt{a+h}(\sqrt{a} + \sqrt{a+h})}\end{aligned}$$

9. Express in terms of the sine and cosine functions and simplify:

$$g(x) = \csc^2(x) + \sec^2(x)$$

Answer:

$$\begin{aligned}
\csc^2(x) + \sec^2(x) &= \frac{1}{\sin^2(x)} + \frac{1}{\cos^2(x)} \\
&= \frac{\cos^2(x) + \sin^2(x)}{(\sin(x)\cos(x))^2} \\
&= \frac{1}{\left(\frac{1}{2}\sin(2x)\right)^2} \\
&= \frac{4}{\sin^2(2x)}
\end{aligned}$$

(Note: $\frac{1}{(\sin(x)\cos(x))^2}$ is also a possible answer, which you can see either by stopping after the second line above, or by using the double angle formula $\sin(2A) = 2\sin(A)\cos(A)$ on the final result.)