Math 135-01, Daily Worksheet 10/7.

(1)

(a) $P(x) = x^3 - 3x^2 + 5x - 7$ satisfies $P(2) = -1$ and $P(3) = 8$. $P(x)$ is a polynomial function, so it is continuous at all real x. Hence by the IVT, since $-1 < 0 < 8$, there is an x in $[2,3]$ with $P(x) = 0$.

(b) If $A > 0$, then $\lim_{x \to \infty} P(x) = +\infty$ and $\lim_{x \to -\infty} P(x) = -\infty$.
If $A < 0$, then $\lim_{x \to \infty} P(x) = -\infty$ and $\lim_{x \to -\infty} P(x) = +\infty$.
(or, you can factor out A, and reduce to the first case).
In both cases, this shows there must be an interval $[a,b]$ such that $P(a) \cdot P(b) < 0$.
then IVT shows $P(x) = 0$ for some x with $a < x < b$.

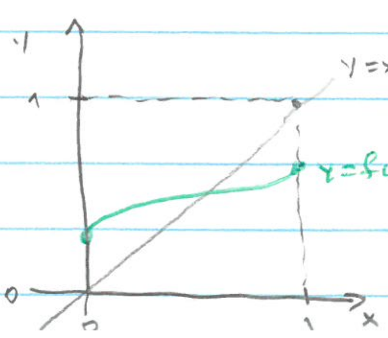
(C) The same reasoning applies to any polynomial of <u>odd</u> degree $d = 1, 3, 5, 7, \ldots$.

(d) We have

x	-3	-2	-1	0	1	2	3
$s(x)$	170	-25	2	-1	2	-25	175

The IVT implies there are roots in each interval $(-3,-2)$, $(-2,-1)$, $(-1,0)$, $(0,1)$, $(1,2)$, $(2,3)$.

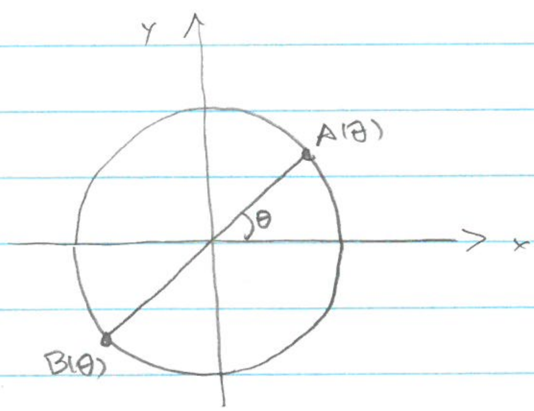
(2)



The given conditions say the graph $y = f(x)$ lies entirely within the "box" $[0,1] \times [0,1]$ for x in $[0,1]$

If $f(0)=0$ or $f(1)=1$, there is nothing to prove. So we may assume $f(0)>0$ and $f(1)<1$. Hence the function $f(x)-x$ changes sign on $[0,1]$: $f(0)-0>0$ and $f(1)-1<0$. Since we assume f is continuous, the same is true for $g(x)=f(x)-x$. Applying the IVT to $g(x)$ on $[0,1]$ gives the desired result.

(3) Temperatures should change continuously from point to point as we go around the circle.



Note $f(\pi) = -f(0)$ since $B(\pi)=A(0)$, $A(\pi)=B(0)$.
Hence IVT implies $f(\theta)=0$ for some $0<\theta<\pi$
since $f(\pi)\cdot f(0) < 0$.