MATH 135 – Calculus 1
Trigonometric Derivatives
October 28, 2019

Background

In today's video, we saw how the addition formulas for $\sin(x)$ and $\cos(x)$, combined with some trigonometric limits we saw back in Chapter 2 of the text lead to the derivative formulas:

$$\frac{d}{dx}\sin(x) = \cos(x) \qquad \text{and} \qquad \frac{d}{dx}\cos(x) = -\sin(x).$$

Questions

(1) For each function, use the appropriate short-cut rules to find the first derivative, and then differentiate again to get the second derivative:

(a) $f(x) = 3\sin(x) + 4\cos(x)$.

Answer: $f'(x) = 3\cos(x) - 4\sin(x)$ and $f''(x) = -3\sin(x) - 4\cos(x)$. (Note that $f''(x) = -f(x)$. The same is true for all linear combinations $f(x) = A\sin(x) + B\cos(x)$, where A, B are constants.)

(b) $g(x) = \cot(x) = \dfrac{\cos(x)}{\sin(x)}$. Your life will be a lot easier here if you simplify the first derivative *before differentiating again* to get $g''(x)$.

Answer:

$$\begin{aligned} g'(x) &= \frac{\sin(x)\cdot(-\sin(x)) - \cos(x)\cdot\cos(x)}{\sin^2(x)} \\ &= \frac{-(\sin^2(x) + \cos^2(x))}{\sin^2(x)} \\ &= \frac{-1}{\sin^2(x)} \qquad \text{(trig identity)} \\ &= -\csc^2(x) \\ \text{Then} \qquad g''(x) &= -2\csc(x)\cdot(-\csc(x)\cot(x)) \qquad \text{(product rule)} \\ &= 2\csc^2(x)\cot(x). \end{aligned}$$

(c) $h(x) = \sin(x)e^x$. Also find the third derivative $h'''(x)$ for this one.

Answer: By the product rule:

$$\begin{aligned} h'(x) &= (\sin(x) + \cos(x))e^x \\ h''(x) &= 2\cos(x)e^x \\ h'''(x) &= 2(\cos(x) - \sin(x))e^x \end{aligned}$$

(2) Consider the graph $y = x - \sin(x)$.

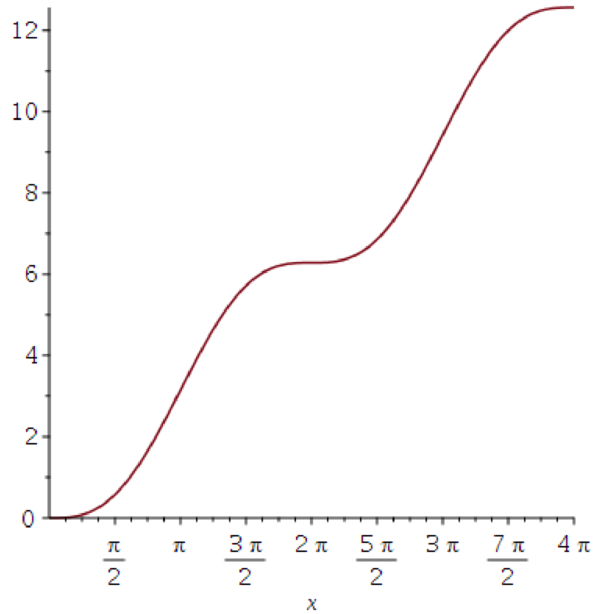

Figure 1: The graph $y = x - \sin(x)$ for $0 \le x \le 4\pi$

(a) Do the tangent lines to this graph ever have a negative slope? Why or why not?

Answer: No, since $\frac{dy}{dx} = 1 - \cos(x)$. This is ≥ 0 for all x.

(b) Do the tangent lines ever have zero slope? Where does that happen?

Answer: Yes, when $cos(x) = 1$. That happens for $x = \pm 2k\pi$ where k is an integer.

(c) Where do the tangent lines have the steepest positive slope? For which x does that happen?

Answer: The largest positive value of the slope is 2, which happens at $x = \pm(2k+1)\pi$ where k is an integer.

(d) Sketch the graph $y = x - \sin(x)$ and check your work with a graphing calculator if you have one (or if one of your classmates can share theirs).

Answer: See graph above for $0 \le x \le 4\pi$: