

Daily Worksheet for 10/21 — Product Rule

(1)

$$(a) \frac{d}{dx} (x^2 + 3x + 1)(x^3 + x + 2)$$

By the product rule:

$$\begin{aligned} &= (x^2 + 3x + 1)(3x^2 + 1) + (x^3 + x + 2)(2x + 3) \\ &= 5x^4 + 12x^3 + 6x^2 + 10x + 7 \end{aligned}$$

Multiplying out, the function =  $x^5 + 3x^4 + 2x^3 + 5x^2 + 7x + 2$

$$\frac{d}{dx} (x^5 + 3x^4 + 2x^3 + 5x^2 + 7x + 2) = 5x^4 + 12x^3 + 6x^2 + 10x + 7 \quad \checkmark$$

$$\begin{aligned} (b) \frac{d}{dx} (x^2 + x + 1)e^x &= (x^2 + x + 1)e^x + e^x(2x + 1) \\ &= (x^2 + 3x + 2)e^x \end{aligned}$$

$$(c) \frac{d}{dx} (e^{2x}) = \frac{d}{dx} (e^x \cdot e^x) = e^x \cdot e^x + e^x \cdot e^x = 2e^{2x}$$

$$(d) \frac{d}{dx} (x^{1/2} + 3x)(x - 4e^x) = (x^{1/2} + 3x)(1 - 4e^x) + (x - 4e^x)(\frac{1}{2}x^{-1/2} + 3)$$

$$\begin{aligned} (2) \text{ (1st stage): } \frac{d}{dx} (x^2 + 3x)[(e^x + x)(x + x^{-2})] \\ &= (x^2 + 3x) \frac{d}{dx} [(e^x + x)(x + x^{-2})] + (e^x + x)(x + x^{-2})(2x + 3) \end{aligned}$$

$$\text{(2nd stage): } = (x^2 + 3x)[(e^x + x)(1 - 2x^{-3}) + (x + x^{-2})(e^x + 1)] + (e^x + x)(x + x^{-2})(2x + 3)$$

$$(3) \frac{d}{dx} f(x)g(x)h(x) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

$$(4) \frac{d}{dx} (f_1(x)f_2(x)\dots f_n(x)) = f_1'(x)f_2(x)\dots f_n(x) + f_1(x)f_2'(x)\dots f_n(x) + \dots + f_1(x)\dots f_{n-1}(x)f_n'(x)$$

that is, the derivative of the product  $f_1(x) \cdots f_n(x)$  is a sum of  $n$  terms. Each has the derivative of one of the  $f_i(x)$  multiplied by the product of the other  $n-1$  factors, undifferentiated.