MATH 135 – Calculus 1
Exam 3 Practice Problems
November 20, 2019

Disclaimers

The actual exam problems may be worded differently and might combine topics shown here. Also, the actual exam will be significantly shorter than this list of practice problems. The purpose of this list is to show the range of topics that might be covered.

I. Find the indicated derivative(s) and simplify.

(A) y' for

$$y = \ln(x)\left(x^7 - \frac{4}{\sqrt{x}}\right)$$

(B) y' for

$$y = \sin^{-1}(e^{2x} + 2)$$

(C) y' for

$$y = \frac{\ln(x+1)}{3x^4 - 1}$$

(D) y' for

$$y = \frac{\sin(x)}{1 + \cos(x)}$$

(E) y' for

$$y = \tan^{-1}(x^2 + x)$$

(F) Find $\frac{dy}{dx}$ using implicit differentiation:

$$xy^2 - 3y^3 + 2x^4 - 4xy = 2$$

(G) Find the equation of the line tangent to the curve from (F) at $(x, y) = (1, 0)$

(H) Find

$$\frac{d}{dx}\left(5x\sqrt{x} - \frac{2}{x^3} + 11x - 4\right)$$

(I)

$$\frac{d}{dt}\left(\frac{t^2e^{3t}}{t^4 + 1}\right)$$

(J)

$$\frac{d^2}{dz^2}\left(\frac{z^2 - 2z + 4}{z^2 + 1}\right)$$

(L)

$$\frac{d}{dx}\left(\sin(x)\left(x^7 - \frac{4}{\sqrt{x}}\right)\right)$$

(M) Find y' (note this is just another way of asking the same question!)

$$y = (e^{2x} + 2)^3$$

(N) Find y' and y''

$$y = \frac{x+1}{3x^4 - 1}$$

(O) Find y'

$$y = \frac{\sin(x)}{1+\cos(x)} + x^2 \cos(x^3 + 3)$$

II. The total cost (in $) of repaying a car loan at interest rate of r% per year is $C = f(r)$.

A) What is the meaning of the statement $f(7) = 20000$?

B) What is the meaning of the statement $f'(7) = 3000$? What are the units of $f'(7)$?

III. The quantity of a reagent present in a chemical reaction is given by $Q(t) = t^3 - 3t^2 + t + 30$ grams at time t seconds for all $t \geq 0$. (Note: For a question like this, I could also give you the plot of the function and ask questions like those below. In this case you need to start from the formula and compute $Q'(t)$; if you were given the graph, you need to make the connection between slopes of tangent lines and signs of $Q'(t)$ visually.)

(A) Over which intervals with $t \geq 0$ is the amount increasing? (i.e. $Q'(t) > 0$) decreasing (i.e. $Q'(t) < 0$)?

(B) Over which intervals is the rate of change of Q increasing? decreasing? decreasing?

IV. Compute the following limits using L'Hopital's Rule or other methods, as appropriate.

(A) $\displaystyle\lim_{x \to 0^+} x^{1/3} \ln(x)$

(B) $\displaystyle\lim_{x \to \infty} \frac{x^3}{e^{2x}}$

(C) $\displaystyle\lim_{x \to 1} \frac{x^2 - 5x + 4}{x^2 - 3x + 2}$

(D) $\displaystyle\lim_{x \to \infty} \left(1 + \frac{4}{x}\right)^x$

V. All parts of this question refer to $f(x) = 4x^3 - x^4$.

(A) Find and classify all the critical points of f using the First Derivative Test.

(B) Over which intervals is the graph $y = f(x)$ concave up? concave down?

(C) Sketch the graph $y = f(x)$.

(D) Find the absolute maximum and minimum of $f(x)$ on the interval $[1, 4]$.

VI. All three parts of this question refer to the function $f(x)$ *whose derivative is plotted in Figure 1. NOTE: This is the graph* $y = f'(x)$ *not* $y = f(x)$.

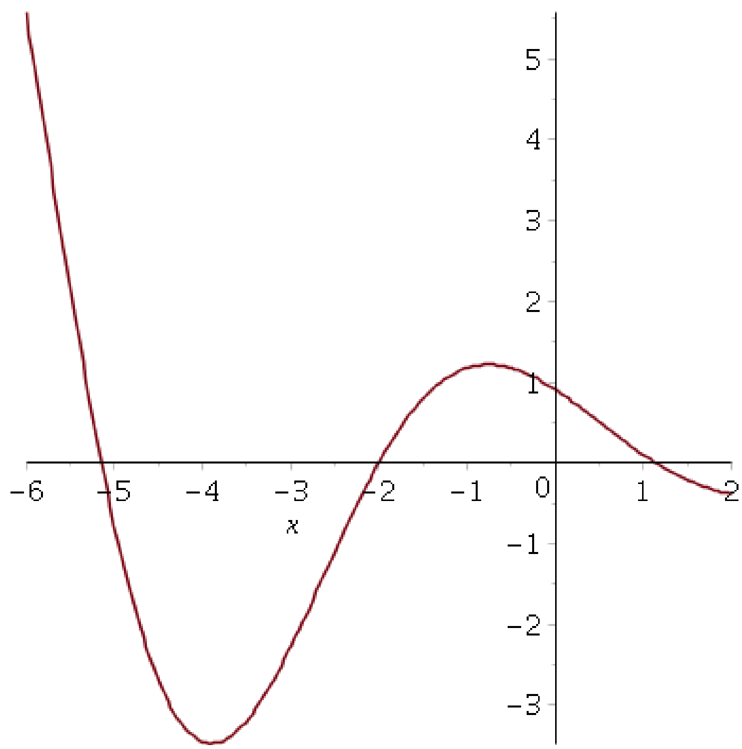

Figure 1: Plot of $y = f'(x)$ for Problem VI

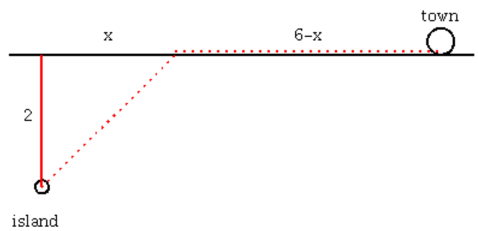

Figure 2: Figure for problem VII.

(A) Give approximate values for all the critical points of $f(x)$ in the interval shown, and say whether f has a local maximum, a local minimum, or neither at each.

(B) Find approximate values for all the inflection points of $f(x)$.

(C) Over which intervals is $y = f(x)$ concave up? concave down?

VII. A town wants to build a pipeline from a water station on a small island 2 miles off the shore of its water reservoir to the town. One possible route is shown dotted in red. The town is 6 miles along the shore from the point nearest the island. It costs \$3 million per mile to lay pipe under the water and \$2 million per mile to lay pipe along the shoreline.

A) Give the cost $C(x)$ of constructing the pipeline as a function of x.

B) Where along the shoreline should the pipeline hit land to minimize the costs of construction?