MATH 135 – Calculus 1
The Derivative Product Rule
October 21, 2019

Background

In today's video, we saw that the derivative of a product of functions is given by

$$(f \cdot g)'(x) = f(x)g'(x) + g(x)f'(x),$$

provided that f and g are each differentiable at x. (In other words, if f, g are differentiable separately, then so is the product, and the derivative of $f \cdot g$ is given by the formula above.) Today, we want to practice with this new derivative rule

Questions

(1) Differentiate by the product rule:

   (a) $(x^2 + 3x + 1)(x^3 + x + 2)$. Then, expand out the product, differentiate using our more basic rules, and compare your answers. They should be equal.

   (b) $(x^2 + x + 1)e^x$.

   (c) e^{2x} (How can you break this up as an $f(x)g(x)$?)

   (d) $(x^{1/2} + 3x)(x - 4e^x)$.

(2) Differentiate using the product rule "in two stages:"

$$(x^2 + 3x)(e^x + x)(x + x^{-2}).$$

(3) Now suppose you have a product $y = f(x)g(x)h(x)$ with *three factors* in general. If you group the terms like this:

$$f(x)g(x)h(x) = f(x) \cdot (g(x)h(x)),$$

explain why

$$y' = f(x)(g(x)h(x))' + (g(x)h(x))f'(x).$$

Now work out $(g(x)h(x))'$ by the product rule and rewrite things to see

$$y' = f(x)g(x)h'(x) + f(x)g'(x)h(x) + f'(x)g(x)h(x).$$

(4) Continuing from (3), what is the pattern here? What if you have a product with any number of factors $y = f_1(x)f_2(x)\cdots f_n(x)$. What will the derivative of that product look like?