MATH 135 – Calculus 1
Practice on Absolute Values, Intervals, Functions
August 31, 2016

Background

Recall from high school mathematics (also see Section 1.1 in Rogawksi/Adams and the video "lecture" for today on Moodle) that the absolute value of a real number is defined as

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

There are several different types of *intervals* in the real number system written as follows:

$$(a, b) = \{x \mid a < x < b\} \tag{1}$$
$$[a, b] = \{x \mid a \leq x \leq b\} \tag{2}$$
$$[a, b) = \{x \mid a \leq x < b\} \tag{3}$$
$$(a, b] = \{x \mid a < x \leq b\}. \tag{4}$$

Today we will practice on some problems related to these ideas.

Questions

1) Real numbers can all be expressed by (possibly infinite or non-repeating) decimal expansions. But exactly what does a finite decimal expansion like $\frac{15}{8} = 1.875$ mean? (Hint: It's really a way of writing the number as a sum of fractions with denominators of a particular form.) What does an infinite decimal expansion like

$$\sqrt{2} = 1.41421356\cdots$$

mean?

2) The set of real numbers x satisfying $|2x + 1| < 4$ is an interval. Write it using the appropriate interval notation as in (1)-(4) above. What would change in your interval notation if we changed $<$ to $\leq$?

3) Write the interval the interval $\left[-\frac{2}{5}, 3\right]$ in the form: "The set of all x such that $|x - c| \leq r$" (What are c, r?)

4) Using the plot of temperature versus time over a 24-hour period at a particular location (on the back of this page) answer these questions:

(a) From the plot, does it appear that the temperature is a *function of time* over this period? Why or why not?

(b) Write the collection of all *temperatures* measured over the 24 hour period as an interval $[a, b]$ for suitable temperature values a, b in degrees Fahrenheit. (There's an unstated assumption behind this question. Can you see what that is? Does it seem reasonable to make that assumption?)
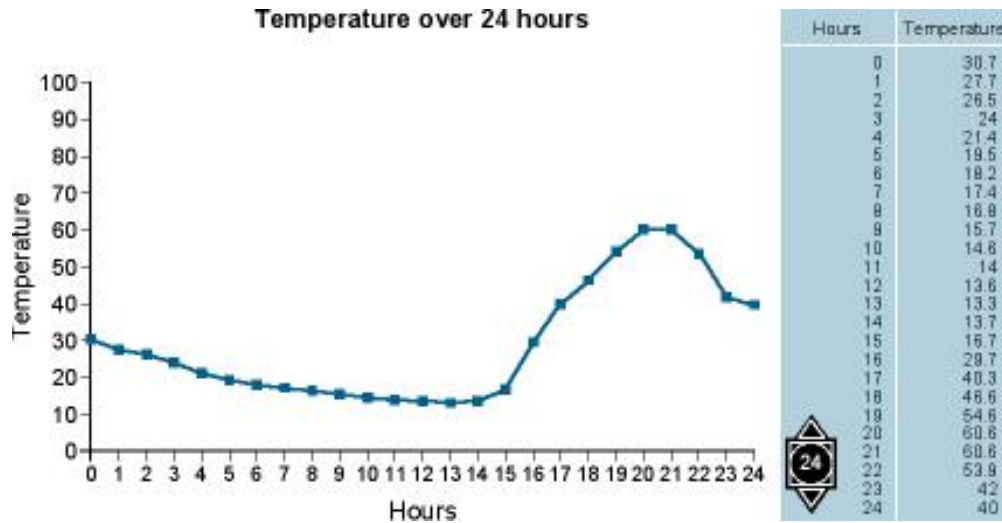
Figure 1: Figure for Question 6

(c) Estimate the interval $[c, d]$ of *times* in hours for which the temperature was 16.8 degrees or less.

(d) At how many different times does it appear that the temperature was exactly 40 degrees F?

Note: The temperature values at the start of each hour are given in the table at the right.