MATH 135 – Calculus 1
More on the Limit Laws
September 26, 2016

Background

Last week we introduced the Limit Laws that show how to compute limits for various combinations of functions when we know each "piece" has a limit separately. Today we will do a few more examples using those.

Questions

(1) Assuming $\lim_{x\to 6} f(x) = 4$, compute the following limits, saying which of the Limit Laws you are using at every step.

   (a) $\lim_{x\to 6}(f(x))^2$.

   (b) $\lim_{x\to 6} \frac{1}{f(x)}$

   (c) $\lim_{x\to 6} x\sqrt{f(x)}$

(2) Assume that $L(a) = \lim_{x\to 0} \frac{a^x-1}{x}$ exists for all real $a > 0$ and that $\lim_{x\to 0} a^x = 1$ for all real $a > 0$.

   (a) Investigate $L(2)$ and $L(3)$ numerically. What are your guesses for those values?

   (b) Explain why $(ab)^x - 1 = a^x(b^x - 1) + (a^x - 1)$.

   (c) Use the result in part (b) and the Limit Laws to show $L(ab) = L(a) + L(b)$.

   (d) Using your answers from part (a), show that the formula from part (c) is reasonable numerically when $a = 2$ and $b = 6$. Note this means you will also need to investigate $L(6)$ numerically.

   (e) What does the equation $L(ab) = L(a) + L(b)$ say about $L(a)$ as a function of a? Do you know other functions with that property?