MATH 135 – Calculus 1
The MVT and Consequences
November 13, 2019

Background

Recall from today's video, recall that we now know a statement called the Mean Value Theorem (MVT):

Theorem 1 *(MVT) Let $f(x)$ be continuous on $[a, b]$ and differentiable on (a, b). Then there exists $c \in (a, b)$ such that*

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

The MVT lets us show in a "rigorous" (i.e. complete and convincing) way that some of the patterns we have discussed intuitively actually always hold. For instance

- If $f'(x) > 0$ on an interval (a, b), then f is increasing on that interval.
- If $f'(x) < 0$ on an interval (a, b), then f is decreasing on that interval.
- If $f'(x) = 0$ on an interval (a, b), then f is constant on that interval.

In addition, we get the following way to determine whether a critical point c of f is a local maximum or local minimum:

Theorem 2 *(First Derivative Test) Let c be a critical point of a function f.*

- *If f' changes sign from positive to negative at c, then $f(c)$ is a local maximum of f.*
- *If f' changes sign from negative to positive at c, then $f(c)$ is a local minimum of f.*
- *If f' does not change sign at c, then $f(c)$ is neither a local maximum nor a local minimum.*

Questions

1. Show that the conclusion of the Mean Value Theorem is true for $f(x) = x^3 - 12x^2 + 21x$ on the interval $[-1, 10]$. That is, find all the $c \in (-1, 10)$ for which $f'(c) = \frac{f(10) - f(-1)}{10 - (-1)}$. There is a plot of the graph $y = f(x)$ on the back of this page. Draw in lines illustrating the conclusion of the Mean Value Theorem.

2. Slim Shady drove through the EZPass lane and got on the Mass Pike at Allston at 11:00am one morning. He drove west to Auburn and exited through the EZPass lane there at 11:37am. The distance between the two exits is 42 miles. Two weeks later, Slim received a speeding ticket from the Mass State Police for $150 in the mail. Should he try to fight this in court, or is the ticket justified? Explain. (Note: The maximum posted speed is 65 miles per hour the whole way.)

3. Now suppose that the graph on the back is $y = g'(x)$ for some other function $g(x)$. What are the critical points of $g(x)$? Classify each of them as a local maximum or local minimum using the First Derivative Test above.

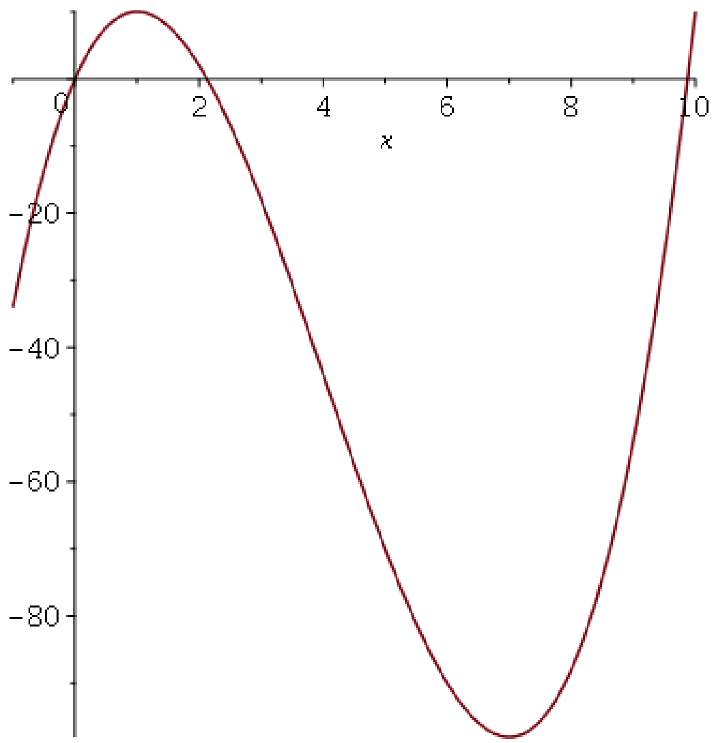

Figure 1: Plot for questions 1 and 3