MATH 135 – Calculus 1
Practice on Linear and Quadratic Functions
September 6, 2019

*Background*

Every line in the plane is described by an equation of the form $Ax + By + C = 0$ for some constants $A, B, C$. If the line is not vertical ($B \neq 0$), then it is the graph of a function. (Why?) Recall that we say a function $f$ is *linear* if $f(x) = mx + b$ for some constants $m, b$. The number $m$ is called the *slope of the line* and the constant $b$ is called the $y$-intercept of the line. *Quadratic functions* have the form $f(x) = Ax^2 + Bx + C$ where $A \neq 0$.

*Questions*

1) Consider lines with equations of the form $2x + cy - 3 = 0$.

   (a) For which value of $c$ does the line contain the point $(x, y) = (1, 2)$?

   (b) For which value of $c$ does the line have slope $-5$?

   (c) Is there any value of $c$ such that the line is horizontal? Why or why not?

   (d) For which value of $c$ is the line perpendicular to the line given by $5x - 3y + 1 = 0$? (Hint: What is true about slopes of perpendicular lines?)

2) The volume $V$ (in liters) of sample of 3 grams of carbon dioxide at 27 degrees Celsius was measured as a function of the pressure $p$ (in atmospheres) with the results in the following table:

| $p$ | 0.25 | 1.00 | 2.50 | 4.00 | 6.00 |
|---|---|---|---|---|---|
| $V$ | 6.72 | 1.68 | 0.67 | 0.42 | 0.27 |

   Is $V$ (approximately) a linear function of $p$? Why or why not? If so, find an approximate formula $V = mp + b$. If not, can you see a equation of a different form for $V$ as a function of $p$?

(3) By the algebraic technique of *completing the square*, every quadratic function $f(x) = x^2 + Bx + C$ (note the coefficient $A = 1$) can be rewritten in the form

$$f(x) = \left(x + \frac{B}{2}\right)^2 + \frac{4C - B^2}{4}$$

   (a) What does this tell us about the relation between the graphs $y = f(x)$ and $y = x^2$? Describe in words.

   (b) Now suppose $A \neq 1$. What is true about the graph $y = Ax^2 + Bx + C$ when $A > 0$? What about when $A < 0$?