MATH 135 – Calculus 1
Review for Final
December 13, 2019

Practice Questions

1. What is the limit definition of the derivative $f'(x)$?
2. Use the definition to compute $f'(x)$ for $f(x) = \sqrt{x-4}$.
3. What is the equation of the tangent line to $y = \sqrt{x-4}$ at $x = 8$?
4. Suppose $f(x)$ is a function such that $f'(x)$ and $f''(x)$ exist and are continuous for all real x. Assume that $f'(x) > 0$ on $(-3,-2)$ and $(2,\infty)$, while $f'(x) < 0$ on $(-\infty,-3)$ and $(-2,2)$. Also assume $f'(-3) = f'(-2) = f'(2) = 0$.
   (a) What does the First Derivative Test tell you about $x = -3$, $x = -2$, and $x = 2$?
   (b) Using the Mean Value Theorem, explain why the equation $f''(x) = 0$ must have a solution in the interval $(-3,-2)$ and another solution in the interval $(-2,2)$.
   (c) Suppose you also know $f(-3) = 4$, $f(-2) = 6$, $f(2) = 0$, and
   $$\lim_{x\to\pm\infty} f(x) = 12.$$
   Sketch a possible graph $y = f(x)$ that satisfies all of these conditions.
5. Let $f(x) = \frac{20x}{x^2-4x+3}$, for which
   $$\begin{aligned} f'(x) &= \frac{-20x^2+60}{(x^2-4x+3)^2} \\ f''(x) &= \frac{40x^3-360x+480}{(x^2-4x+3)^3} \end{aligned}$$
   (a) Where does $y = f(x)$ have vertical asymptotes? Does it have a horizontal asymptote? If so, where?
   (b) What are
   $$\lim_{x\to 1^-} f(x) \text{ and } \lim_{x\to 1^+} f(x)?$$
   What about
   $$\lim_{x\to 3^-} f(x) \text{ and } \lim_{x\to 3^+} f(x)?$$
   (c) Does $f(x)$ have any critical points? Where are they located? What are the critical values?
   (d) What is the concavity of the graph $y = f(x)$ on the interval $(3,\infty)$?
   (e) Sketch the graph $y = f(x)$.