MATH 135 Calculus 1
Practice on Inverse Functions
September 11, 2019

Background

If a function f from a domain D to a range R has the one-to-one property (that is, each y in the range R is $y = f(x)$ for exactly one x in the domain D), then we can define a second function f^{-1} with domain R and range D such that

$$x = f^{-1}(y) \text{ exactly when } y = f(x).$$

This is called the inverse function of f. As we said in todays video, CAUTION: This is not the same as the function defined by $\frac{1}{f(x)}$ (!) We want to work with a few examples of this idea today.

Questions

1) Figure 19 from page 39 in our textbook is copied on the back of this page.

   (a) Which of the graphs there define functions with the one-to-one property?

   (b) For the ones that do not have that property, how could we restrict the domain to get an inverse function? Is there just one way to do that, or is there more than one?

   (c) Recall from Video 1.5 that there is a geometric way to get the graph of an inverse function from the graph of a one-to-one function. Draw the graphs of inverse functions for each of the ones from part (a) and then for each restricted domain function from part (b).

2) For this problem explain why the given function has the one-to-one property. Then find a formula for the inverse function by setting up an equation $y = f(x)$ and solving for x as a function of y. As a last step, swap the variables to write the inverse function as a function of x[1]. In each case state what the domain of the inverse function is.

   (a) $f(x) = 7x + 3$

   (b) $f(x) = \dfrac{1}{x+1}$

   (c) $f(x) = x^2 + 4x + 5$ on the domain $D = [-2, +\infty)$.

[1] As some of you may know, you can also swap the variables first then solve for y as a function of x – you'll get to the same place in the end. If you're more comfortable doing it that way, by all means do.