MATH 135 – Calculus 1
The Derivative of a Function
October 9 and 11, 2019

Initial Background

We are now ready to begin Chapter 3 in our textbook. In the videos for today's class, we introduced the *derivative* of a function f at $x = a$ in the domain of f:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}, \tag{1}$$

$f'(a)$ will give the slope of the tangent line to the graph $y = f(x)$ at the point $(a, f(a))$. If x represents time, and $f(x)$ is a position, then $f'(a)$ would be the instantaneous velocity.

All the techniques we learned in Chapter 2 for computing indeterminate form limits were, in fact, *set up to compute the limits giving* $f'(a)$(!) To start, let's practice (and review) some of those techniques!

Questions

(1) *Using the limit definition* from (1) above, compute $f'(a)$ for $f(x) = x^3 + 2x + 1$–the derivative at a general $x = a$ for $f(x) = x^3 + 2x + 1$. Use your result to find the equation of the tangent line to the graph $y = x^3 + 2x + 1$ at the point $(1, 4)$.

(2) Compute $f'(a)$ for $f(x) = \sqrt{x+1}$–the derivative at a general $x = a$ for $f(x) = \sqrt{x+1}$. Here there is a restriction on which a "work." What is that restriction? Does this make sense, thinking of the graph $y = \sqrt{x+1}$? (Note: this is part of the parabola with equation $x = y^2 - 1$.) Use your result to find the equation of the tangent line to the graph $y = \sqrt{x+1}$ at the point $(3, 2)$.

Further Background

In today's videos we also introduced the idea of deriving formulas for $f'(x)$ for a general x, and considering f' as a new function in its own right. We introduced these "shortcut formulas:"

(a) If $f(x) = x^n$ for any number n, then $f'(x) = nx^{n-1}$.

(b) If $f(x) = e^x$, then $f'(x) = e^x$ (yes, the same function!)

(c) If $f'(x)$ exists, then so does $(kf)'(x)$ and $(kf)'(x) = kf'(x)$.

(d) If $f'(x)$ and $g'(x)$ both exist, then so does $(f+g)'(x)$, and $(f+g)'(x) = f'(x) + g'(x)$.

Additional Questions

(3) Verify the formula in (a) for $f(x) = \frac{1}{x^2} = x^{-2}$. That is, you want to show $f'(x) = -2x^{-3}$ using the limit definition of $f'(x)$.

(4) Using the shortcut rules above, find the derivatives of the following functions:

(a) $f(x) = x^5 + 2x^{3/2} + 4x^{-3}$

(b) $g(x) = x^{1/2} + 3e^x$

(c) $h(x) = e^\pi$ (be careful!)