MATH 134 – Calculus with Fundamentals 2
Practice Day on computing areas between curves via integrals
February 14, 2018

*Background*

As we have seen in some examples, if $f(x) \geq g(x)$ for all $x$ in $[a, b]$ then the area between the graphs $y = g(x)$ and $y = f(x)$ can be computed as

$$\text{Area} = \int_a^b \text{top} - \text{bottom}\; dx = \int_a^b f(x) - g(x)\; dx. \quad (1)$$

If the curves cross somewhere in the integral, so that the equations of the top and bottom change, then the required area can be written using the absolute value as

$$\text{Area} = \int_a^b |f(x) - g(x)|\; dx.$$

But in practice, an integral this would always be computed by determining the points where the curves cross and setting up several integrals of the form (1).

*Questions*

For each problem,

(i) Determine whether the curves cross and where they do if so. (For this step and the next, it often helps to sketch the graphs to see this.)

(ii) Determine which function is "on top" and which is "on the bottom" over each subinterval

(iii) Set up appropriate integrals like the one in (1) for each part and compute each one

(iv) Add the integrals from (iii).

1. Determine the area between $y = x^2$ and $y = -x + 2$ for $x$ with $0 \leq x \leq 2$.

2. Determine the area between $y = 4 - x^2$ and $y = -14 + x^2$ for $x$ with $1 \leq x \leq 5$.

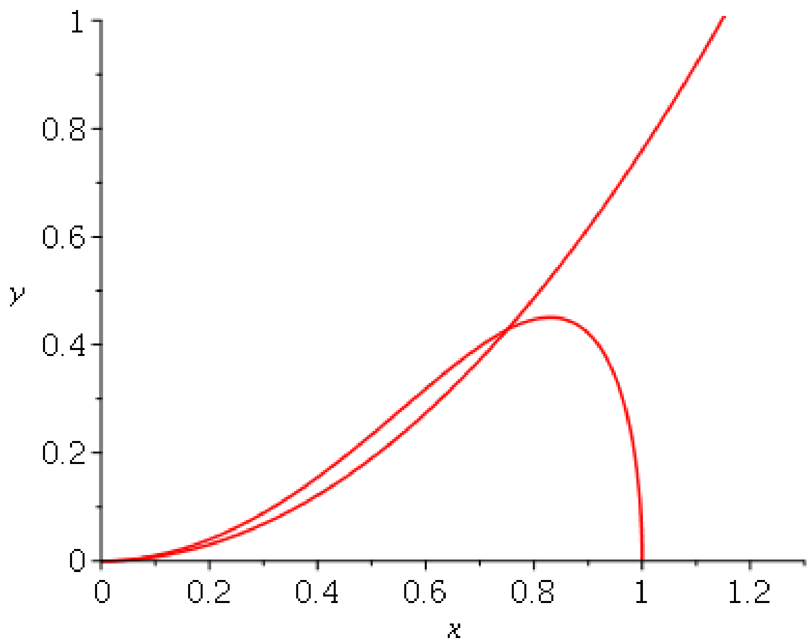


Figure 1: The region for problem 4.

3. Determine the area of *the bounded region* between $y = \frac{10}{x}$ and $y = -x + 11$.

4. The region between $y = x^2\sqrt{1 - x^3}$ and $y = \frac{\sqrt{37}x}{8}$ is plotted in Figure 1 above (it's the tiny curved "needle-shaped" region between the graphs; the parabola is the bottom curve the whole way). Find its area.