MATH 134 – Calculus with Fundamentals 2
Practice Day on Properties of $\int_a^b f(x)\ dx$
January 29, 2018

Background

We have now introduced the definite integral $\int_a^b f(x)\ dx$ and discussed:

- the interpretation of of this number as a *signed area*,
- the *interval additivity property*: If f is integrable on $[a, b]$ and $a \le c \le b$, then
$$\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$$
- the *comparison property*: If f is integrable on $[a, b]$ and $m \le f(x) \le M$ for all x in $[a, b]$, then
$$m \cdot (b - a) \le \int_a^b f(x)\ dx \le M \cdot (b - a).$$

Questions Let $f(x)$ be the function on the interval $[0, 6]$ plotted on the back. Thinking of the definite integral as *signed area*, answer these questions.

(1) What are $\int_0^4 f(x)\ dx$, $\int_4^6 f(x)\ dx$, $\int_0^6 f(x)\ dx$?

(2) From your answers to part (1) verify that $\int_0^6 f(x)\ dx = \int_0^4 f(x)\ dx + \int_4^6 f(x)\ dx$.

(3) What is $\int_5^6 f(x)\ dx$? How can you tell?

(4) Let m be the minimum value of $f(x)$ on $[3, 5]$ and let M be the maximum value of $f(x)$ on $[3, 5]$ (use the information from the graph). Find m, M and verify that
$$m \cdot (5 - 3) \le \int_3^5 f(x)\ dx \le M \cdot (5 - 3)$$

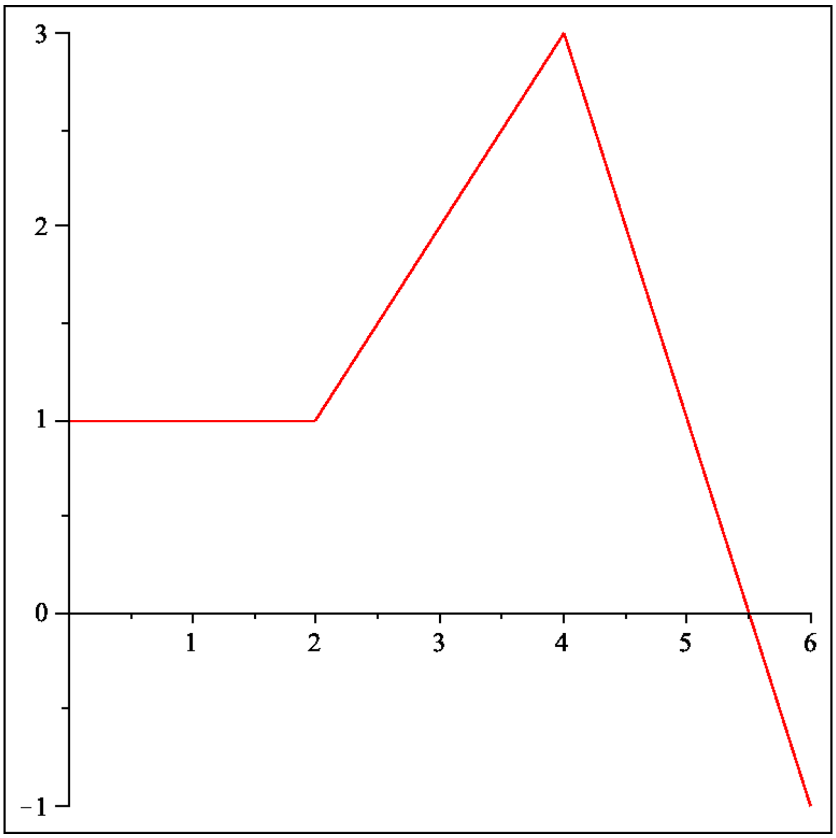

Figure 1: The region