MATH 134 – Calculus with Fundamentals 2
Second Practice on Improper Integrals
April 4, 2018

Background

Whenever $a = -\infty$ or $b = +\infty$ or both, the integral

$$\int_a^b f(x)\ dx$$

is also said to be an *improper integral*. As in the cases we saw yesterday, these new improper integrals are also handled by *taking limits of "ordinary" integrals*.

- We say $\int_a^\infty f(x)\ dx$ *converges* if the limit

  $$\lim_{b\to\infty} \int_a^b f(x)\ dx$$

  exists and we say the integral *diverges* otherwise.

- We say $\int_{-\infty}^b f(x)\ dx$ *converges* if the limit

  $$\lim_{a\to-\infty} \int_a^b f(x)\ dx$$

  exists and we say the integral *diverges* otherwise.

- We say $\int_{-\infty}^\infty f(x)\ dx$ *converges* if, splitting at any finite value c (often at $c = 0$), *both of the limits* here:

  $$\lim_{a\to-\infty} \int_a^c f(x)\ dx + \lim_{b\to+\infty} \int_c^b f(x)\ dx$$

  exist and we say the integral *diverges* otherwise. (Note this says that if either one of the two limits does not exist, then the integral diverges.)

Questions

For each of the following,

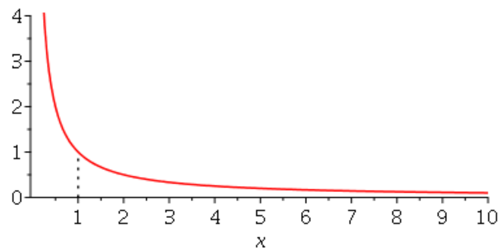

Figure 1: The region for problem 4.

(i) Note which of the limits of integration are infinite.

(ii) Set up the appropriate limit(s) and evaluate.

(iii) Say whether the integral converges or diverges.

(1) $\displaystyle\int_{-\infty}^{2} e^{10x}\, dx$

(2) $\displaystyle\int_{4}^{\infty} \frac{1}{x^2+16}\, dx$

(3) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2+16}\, dx$

(4) Integrals over infinite intervals can lead to somewhat counterintuitive statements. For example, explain why

  (a) The area between the graph $y = \frac{1}{x}$ and the x-axis for $x \geq 1$ is not finite, but

  (b) When the area from part (a) is rotated about the x-axis, it forms a solid of revolution of finite volume(!)