MATH 134 – Calculus with Fundamentals 2
First Practice on Improper Integrals
April 3, 2018

Background

When $[a, b]$ is a finite interval, the integral

$$\int_a^b f(x)\ dx$$

is said to be *improper* whenever $f(x)$ has one or more *discontinuities* in the interval $[a, b]$. These can happen at the endpoints a or b, or at some c in (a, b). In all cases, improper integrals are handled by *taking limits of "ordinary" integrals.* For the purposes of this discussion, let us suppose that $f(x)$ has only one point of discontinuity in the interval.

- If f is discontinuous at a, then we say $\int_a^b f(x)\ dx$ *converges* if the limit

  $$\lim_{c\to a^+} \int_c^b f(x)\ dx$$

  exists and we say the integral *diverges* otherwise.

- If f is discontinuous at b, then we say $\int_a^b f(x)\ dx$ *converges* if the limit

  $$\lim_{c\to b^-} \int_a^c f(x)\ dx$$

  exists and we say the integral *diverges* otherwise.

- If f is discontinuous at d in (a, b), then we say $\int_a^b f(x)\ dx$ *converges* if *both of the limits* here:

  $$\lim_{c\to d^-} \int_a^c f(x)\ dx + \lim_{c\to d^+} \int_c^b f(x)\ dx$$

  exist and we say the integral *diverges* otherwise. (Note this says that if either one of the two limits does not exist, then the integral diverges.)

Questions

For each of the following,

(i) Say why the integral is improper by locating any discontinuities of the function.

(ii) Set up the appropriate limit(s) and evaluate.

(iii) Say whether the integral converges or diverges.

1. $\displaystyle\int_1^2 \frac{1}{\sqrt[3]{x-1}}\,dx$

2. $\displaystyle\int_0^2 \frac{1}{x^2-5x+6}\,dx$

3. $\displaystyle\int_1^3 \frac{1}{x^2-4x+4}\,dx$