MATH 134 – Calculus with Fundamentals 2
Practice Day on computing volumes of solids via integrals
February 20, 2018

Background

As we have seen in some examples, let the region R be defined by $0 \leq y \leq f(x)$ for all x in $[a, b]$. Then the volume of the solid of revolution obtained by rotating R about the x-axis is given by

$$\text{Volume} = \int_a^b \pi(f(x))^2 \, dx. \tag{1}$$

Today we want to practice on using (1) to set up and compute integrals for volumes of some regions of this type.

Questions

For each problem,

(i) Set up the volume integral

(ii) Evaluate it using the FTC part I.

1. $f(x) = x^4$, $[a, b] = [0, 1]$.
2. $f(x) = \sec(x)$, $[a, b] = [0, \pi/4]$.
3. $f(x) = \sqrt{x} \cdot (1 - x^2)^{1/4}$, $[a, b] = [0, 1]$.
4. $f(x) = \sin(x)$, $[a, b] = [0, \pi]$. For this one you will want to use a trigonometric identity to rewrite your function before integrating:

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

(The reason this works is because of two other trig identities:

$$\cos^2(x) + \sin^2(x) = 1$$
$$\cos^2(x) - \sin^2(x) = \cos(2x).$$

Subtract the second from the first and solve for $\sin^2(x)$ to see the identity above.)