4, 4, 8 = 16 total

MATH 134 Problem Set 8, Part B Solutions

Section 9.2

17. (b) From the differential equation in part,

$P'(t) = 0.09\,P(t) - N$ so $P(t) = \frac{N}{.09} + Ce^{.09t}$

(see Equation 7 in the text). If $N = 1200$,

$P(t) = 13333.33 + Ce^{.09t}$ and setting $t=0$,

$10000 = 13333.33 + C$, so $\boxed{C = -3333.33}$. (1)

then we want to solve $0 = 13333.33 - 3333.33e^{.09t}$

to see how long it takes to pay off the loan,

$e^{.09t} = \frac{13333.33}{3333.33} \doteq 4$

so $t = \frac{\ln(4)}{.09} \doteq 15.4$ years (1)

(c) If $N = \$800.00$, $P(t) = 8888.89 + Ce^{(.09)t}$. So

setting $t=0$ again $10000 = 8888.89 + C$ gives

$c = 1111.11$ (1). then $8888.89 + 1111.11e^{(.09)} = P(t)$

and $P(t) = 0$ has <u>no</u> solution (1). (the

loan is <u>never</u> paid off because $N = \$800$ is

to small to ever "overcome" the accumulated

interest.)

18. As in problem 17, $P'(t) = .05\,P(t) - N$

so $P(t) = \frac{N}{.05} + Ce^{.05t}$ (1). $P(0) = 18000$

implies

$C = 18000 - \frac{N}{.05}$

Solve for N to make

$$0 = P(5) = \frac{N}{.05} + \left(18000 - \frac{N}{.05}\right) e^{(.05).5} \quad 1$$

$$\Rightarrow \quad 0 = N + (900 - N)\, e^{(.05)5} \qquad \text{(mult. by .05)}$$

$$\doteq N + (900 - N)(1.284)$$

$$= 1155.62 - 0.284N$$

$$\therefore \boxed{N = \$4069.09} \qquad \left(= \frac{1155.62}{0.284}\right) \; 2$$

51 (a) At the end of the N^{th} day, the amount present is

$$R = \underbrace{De^{-k}}_{\text{from most recent dose}} + \underbrace{De^{-2k}}_{\text{dose previous day}} + \cdots + \underbrace{De^{-Nk}}_{N \text{ days previously}} \quad 1$$

$$= (De^{-k}) \big(\underbrace{1 + e^{-k} + e^{-2k} + \cdots + e^{-(N-1)k}}_{\text{a finite geom. series}} \big)$$

$$= De^{-k} \cdot \frac{1 - e^{-Nk}}{1 - e^{-k}}. \quad 1$$

Over "an extended period" can be interpreted as the limiting value as $N \xrightarrow{1} \infty$. $\lim_{N \to \infty} (e^{-k})^N = 0$, so this gives

$$\lim_{N \to \infty} R = \lim_{N \to \infty} \frac{De^{-k}(1 - (e^{-k})^N)}{1 - e^{-k}}$$

$$= \boxed{\frac{De^{-k}}{1 - e^{-k}}}. \quad 1$$

(b) this is exactly the same idea as in (a), but look at N = number of doses administered

$$\lim_{N\to\infty} De^{-kt}\left(1+e^{-kt}+e^{-2kt}+\cdots+e^{-(N-1)kt}\right)$$

$$= \lim_{N\to\infty} \frac{De^{-kt}\left(1-(e^{-kt})^N\right)}{1-e^{-kt}} \quad 2$$

$$= \frac{De^{-kt}}{1-e^{-kt}} \quad \text{since } \lim_{N\to\infty}(e^{-kt})^N = 0.$$

(c) If S is the largest safe level, we want each new dose D not to take the total level over S: $D+R \le S$.

$$D+R = D+\frac{De^{-kt}}{1-e^{-kt}} = \frac{D}{1-e^{-kt}}$$

so $\dfrac{D}{1-e^{-kt}} = S$ when $\dfrac{D}{S} = 1-e^{-kt}$

$$1-\frac{D}{S} = e^{-kt} \quad \text{(solve for } t\text{)}$$

$$\boxed{t = \frac{-\ln\left(1-\frac{D}{S}\right)}{k}} \quad 2$$

(Larger t would make R smaller, so any

$t \ge \dfrac{-\ln\left(1-\frac{D}{S}\right)}{k}$ is ok.)