2, 6, 2, 4, 2 ~~total~~

MATH 134, Problem Set 7, part B Solutions

§ 7.8 /

2. We need $\int_0^4 c\,x(4-x)\,dx = 1$, so

$$c\int_0^4 4x - x^2\,dx = 1$$

$$c\cdot\left[2x^2 - \frac{x^3}{3}\right]_0^4 = 1$$

$$c\cdot\left(32 - \frac{64}{3}\right) = 1$$

$$\boxed{c = \frac{3}{32}}$$ 1

Then $P(3 \le X \le 4) = \int_3^4 \frac{3}{32}(4x - x^2)\,dx$

$$= \frac{3}{32}\left(2x^2 - \frac{x^3}{3}\right)\Big|_3^4$$

$$= \frac{3}{32}\left(\frac{32}{3} - 9\right)$$

$$= \boxed{\frac{5}{32}}$$ 1

20. (a) $P(128 \le X \le 150) = P\left(\frac{128-102}{48} \le Z \le \frac{150-102}{48}\right)$

$$\doteq P(.52 \le Z \le 1)$$

$$\doteq .3413 - .1958 \quad \text{(from tables)}$$

$$= \boxed{.1455}$$ 2

(Can also be expressed as 1 $F(1) - F(.52)$) — give full credit even if they don't say this

(b) This would be 1 $\int_{120}^{\infty} \frac{1}{\sqrt{2\pi(48)^2}}\, e^{-(x-102)^2/(2\cdot(48)^2)}\,dx$

This is $P(X > 120) = P(Z > \frac{120-102}{48}) = P(Z > .375)$
$= .5000 - P(0 \leq Z \leq .375)$. In the table
we find $P(0 \leq Z \leq .37) \doteq .1443$ and $P(0 \leq Z \leq .38)$
$\doteq .1480$. Averaging these, we have

It's ok if they just use one of these, rather than averaging

$$P(0 \leq Z \leq .375) \doteq \frac{.1443 + .1480}{2} \doteq .14615$$

2

so $P(X > 120) \doteq .5000 - .14615 \doteq \boxed{.35385}$

21. $P(X < 31) = P\left(\frac{X-32}{.4} < \frac{31-32}{.4}\right)$

$= P(Z < -2.5)$

$= P(Z > +2.5)$ (by symmetry)

$= .5000 - .4938$ (from table)

$= \boxed{.0062}$ 2

(integral is $\int_{-\infty}^{31} \frac{1}{\sqrt{2\pi}\cdot(.4)} e^{-(x-32)^2/2(.4)^2}\, dt$.)

§ 4.5/47 $\lim_{x\to 1} (1+\ln x)^{\frac{1}{x-1}}$ is a 1^{∞} form

1

We take ln, then rearrange to apply L'Hopital:

$$\lim_{x\to 1} \ln\left((1+\ln x)^{\frac{1}{x-1}}\right) = \lim_{x\to 1} \frac{1}{x-1} \ln(1+\ln x) \qquad 0\cdot\infty$$

$$= \lim_{x\to 1} \frac{\ln(1+\ln x)}{x-1} \qquad \frac{0}{0}$$ 1

$$= \lim_{x\to 1} \frac{\frac{1}{1+\ln x}\cdot\frac{1}{x}}{1} \qquad \text{(L'H.)}$$ 1

$= 1.$

therefore, $\lim_{x \to 1} (1 + \ln x)^{\frac{1}{x-1}} = e^1 = \boxed{e}$ 1

§ 5.9 | 54 With continuous compounding the balance at time t is $P = P_0 e^{Rt}$ (P_0 = initial deposit). We find the doubling time by setting this equal to $2P_0$ and solving for t:

$$\cancel{P_0} e^{Rt} = 2\cancel{P_0}$$

$$Rt = \ln(2)$$

$$\boxed{t = \frac{\ln(2)}{R}}$$ 1

If R is expressed as $\frac{r}{100}$ (e.g. $\overset{r}{5}\% = \frac{5}{100} = \overset{R}{.05}$) then this equals

$$t = \frac{100 \ln(2)}{r}$$

Since $100 \ln(2) \doteq 69.3$, this is close to $\frac{70}{r}$. 1