2, 4, 4, 4, 4 18 points total

MATH 134, Problem Set 5, part B solutions

Section 7.2 /

63. We have
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\sin(4x) = 2\sin(2x)\cos(2x)$$
$$= 4\sin(x)\cos^3(x) - 4\sin^3(x)\cos(x)$$
$$= 4\sin(x)(1-\sin^2(x))\cos(x) - 4\sin^3(x)\cos(x)$$
$$= 4\sin(x)\cos(x) - 8\sin^3(x)\cos(x).$$
1

Hence, substituting in the first form

$$\frac{1}{32}\left(12x - 8\sin(2x) + \sin(4x)\right) + C$$

$$= \frac{3x}{8} - \frac{1}{2}\sin(x)\cos(x) + \frac{1}{32}\left(4\sin(x)\cos(x) - 8\sin^3(x)\cos(x)\right) + C$$

$$= \frac{3x}{8} - \frac{3}{8}\sin(x)\cos(x) - \frac{1}{4}\sin^3(x)\cos(x) + C$$
1

this is the second form.

65. the volume is:

$$V = \int_0^{\pi} \pi(\sin(x))^2\,dx$$ 2 (disk cross-sections)

$$= \pi\left[-\frac{\sin(x)\cos(x)}{2} + \frac{x}{2}\right]\Bigg|_0^{\pi}$$ 1 (using SC1)

$$= \pi\cdot\left[0 + \frac{\pi}{2} - 0\right]$$

$$= \boxed{\frac{\pi^2}{2}}$$ 1

79. $\int \sec^m(x)\,dx \qquad (m \neq 1)$

Let $u = \sec^{m-2}(x)\,dx \qquad dv = \sec^2(x)\,dx \implies v = \tan(x)$

$du = (m-2)\sec^{m-3}(x)\cdot\sec(x)\tan(x)\,dx$ 2

$\boxed{du = (m-2)\sec^{m-2}(x)\tan(x)\,dx}$

Then by parts,

$$\int \sec^m(x)\,dx = \sec^{m-2}(x)\tan(x) - \int (m-2)\sec^{m-2}(x)\underbrace{\tan^2(x)}\,dx$$

trig identity 1

$$= \sec^{m-2}(x)\tan(x) - (m-2)\int \sec^{m-2}(x)\left(\sec^2(x)-1\right)dx$$

$$\int \sec^m(x)\,dx = \sec^{m-2}(x)\tan(x) \underbrace{- (m-2)\int \sec^m(x)\,dx}_{\text{add to both sides}} + (m-2)\int \sec^{m-2}(x)\,dx$$

1

$$\implies (m-1)\int \sec^m(x)\,dx = \sec^{m-2}(x)\tan x + (m-2)\int \sec^{m-2}(x)\,dx$$

so $$\boxed{\int \sec^m(x)\,dx = \frac{\sec^{m-2}(x)\tan(x)}{m-1} + \frac{m-2}{m-1}\int \sec^{m-2}(x)\,dx}$$

Section 7.3 /

34. $\displaystyle\int \frac{dx}{(x^2+a)^2}$ Let $x = \sqrt{a}\tan\theta \qquad dx = \sqrt{a}\sec^2\theta\,d\theta$

1

$$= \int \frac{\sqrt{a}\sec^2\theta\,d\theta}{(a\tan^2\theta + a)^2}$$

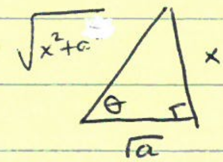
$$= \int \frac{\sqrt{a}\sec^2\theta\,d\theta}{a^2(\tan^2\theta+1)^2}$$

1

$$= \int \frac{\sqrt{a}\sec^2\theta\,d\theta}{a^2\sec^4\theta} = \frac{1}{a^{3/2}}\int \frac{1}{\sec^2\theta}\,d\theta = \frac{1}{a^{3/2}}\int \cos^2\theta\,d\theta$$

using the trig reduction SC 2, this becomes

$$= \frac{1}{a^{3/2}}\left[\frac{\sin\theta\cos\theta}{2} + \frac{\theta}{2} \right] + C$$



1

Converting back,

$$= \frac{1}{a^{3/2}} \cdot \frac{x}{\sqrt{x^2+a}} \cdot \frac{\sqrt{a}}{\sqrt{x^2+a}} \cdot \frac{1}{2} + \frac{1}{2a^3} \tan^{-1}\left(\frac{x}{\sqrt{a}}\right) + C$$

$$= \frac{1}{2a} \frac{x}{x^2+a} + \frac{1}{2a^{3/2}} \tan^{-1}\left(\frac{x}{\sqrt{a}}\right) + C$$

1

36. Completing the square, $12x - x^2 = 36 - (x-6)^2$

So $\displaystyle\int \frac{dx}{\sqrt{36-(x-6)^2}}$

Let $x - 6 = 6\sin\theta$

$dx = 6\cos\theta\, d\theta$

then $\frac{x-6}{6} = \sin\theta$

2

$\theta = \sin^{-1}\left(\frac{x-6}{6}\right)$

$$= \int \frac{6\cos\theta\, d\theta}{\sqrt{36(1-\sin^2\theta)}}$$

$$= \int \frac{\cancel{6}\,\cancel{\cos\theta}\, d\theta}{\cancel{6}\,\cancel{\cos\theta}}$$

$$= \int d\theta$$ 1

$$= \theta + C$$

$$= \boxed{\sin^{-1}\left(\frac{x-6}{6}\right)} + C.$$

1