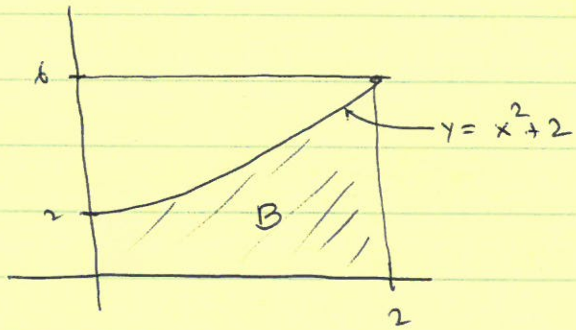
4, 4, 4, 6   18 points total

MATH 134 — Problem Set 4, part B Solutions

Section 6.4



42. Rotating B about the line $x = -3$. The most convenient method is cylindrical shells:

$$V = \int_0^2 2\pi (x+3)(x^2+2)\,dx \quad 2$$

$$= 2\pi \int_0^2 x^3 + 3x^2 + 2x + 6\,dx$$

$$= 2\pi \left[\frac{x^4}{4} + x^3 + x^2 + 6x \right]_0^2$$

$$= 2\pi \left[4 + 8 + 4 + 12 - 0 \right]$$

$$= \boxed{56\pi} \quad 2$$

(this can also be done by slices:

$$V = \int_0^2 \pi \cdot 5^2 - \pi \cdot 3^2\,dy + \int_2^6 \pi \cdot 5^2 - \pi \big(\underbrace{\sqrt{y-2}+3}_{\sqrt{y-2}+3}\big)^2\,dy \Big)$$

OK if they set it up this way!

44. Rotating B about the x-axis: Disc cross-section

$$V = \int_0^2 \pi (x^2+2)^2\,dx \quad = \pi \int_0^2 x^4 + 4x^2 + 4\,dx$$
$\quad 2$

$$= \pi \left[\frac{x^5}{5} + \frac{4x^3}{3} + 4x \right]_0^2$$

$$= \boxed{\frac{376\pi}{15}} \quad 2$$

46. Rotating B about the line $y = 8$ gives "washer" cross-sections

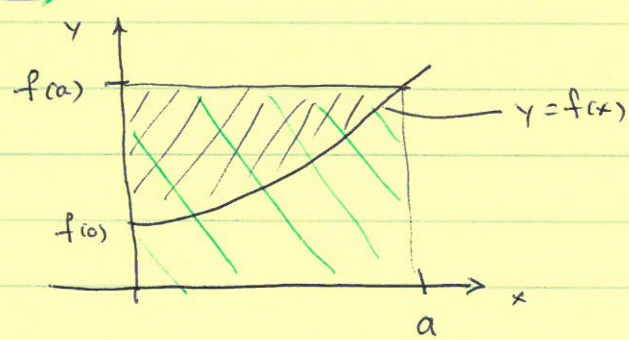
$$V = \int_0^2 \pi (8)^2 - \pi \left(8 - (x^2+2) \right)^2 \, dx \quad 2$$

$$= \pi \int_0^2 28 + 12x^2 - x^4 \, dx$$

$$= \pi \left[28x + 4x^3 - \frac{x^5}{5} \right]_0^2$$

$$= \boxed{\frac{408\pi}{5}} \quad 2$$

Section 7.1 / 88



Method 1 (parts) In $\int_0^a f(x)\,dx$, let $u = f(x)$, $dv = dx$ then $du = f'(x)\,dx$ and $v = x$, so 1

$$\int_0^a f(x)\,dx = x f(x) \Big|_0^a - \int_0^a x f'(x)\,dx$$

$$= a f(a) - \int_0^a x f'(x)\,dx. \quad 1$$

Method 2: the area of the green-shaded rectangle is $a \cdot f(a)$, and $\int_0^a f(x)\,dx$ gives the area between $y = f(x)$ and the x-axis. Hence we just need to show that $\int_0^a x\, f'(x)\,dx$ gives the shaded (black) area from the problem. To see this, let

$$\begin{cases} y = f(x) \iff x = f^{-1}(y) \\ dy = f'(x)\,dx \end{cases}$$ /1

and substitute:

$$\int_0^a x\, f'(x)\,dx = \int_{f(0)}^{f(a)} f^{-1}(y)\,dy$$ /1

this integral computes the area between $x = f^{-1}(y) \iff y = f(x)$ and the y-axis, for y from $f(0)$ to $f(a)$. //