4, 4, 4, 4 16 total points

MATH 134 Problem Set 3, part B Solutions

5.7/109

$$\int_a^b \frac{1}{x}\,dx = \ln x \Big|_a^b = \ln b - \ln a = \ln(b/a) = \int_1^{b/a} \frac{1}{x}\,dx,$$

since $\int_1^{b/a} \frac{1}{x}\,dx = \ln x \Big|_1^{b/a} = \ln(b/a) - \ln(1)$, but $\ln(1) = 0$.

2

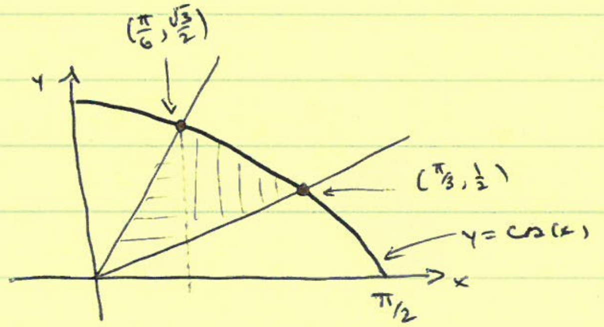
The areas shown all have the form $\int_{2^k}^{2^{k+1}} \frac{1}{x}\,dx$ for some $k \geq 0$. By the first part of this solution

$$\int_{2^k}^{2^{k+1}} \frac{1}{x}\,dx = \int_1^{2^{k+1}/2^k} \frac{1}{x}\,dx = \int_1^2 \frac{1}{x}\,dx$$

2

Hence all the given areas are equal to $\int_1^2 \frac{1}{x}\,dx = \ln 2$.

6.1/19



Split the region at $x = \pi/6$. In the part to the left of $x = \frac{\pi}{6}$,

$y_{top} = \frac{\sqrt{3}/2}{\pi/6}x = \frac{3\sqrt{3}\,x}{\pi}$ and $y_{bottom} = \frac{1/2}{\pi/3}x = \frac{3x}{2\pi}$

To the right, $y_{top} = \cos x$, $y_{bottom} = \frac{3x}{2\pi}$. So

$$\text{Area} = \left.\int_0^{\pi/6} \frac{3\sqrt{3}\,x}{\pi} - \frac{3x}{2\pi}\,dx + \int_{\pi/6}^{\pi/3} \cos(x) - \frac{3x}{2\pi}\,dx\right]^{2}$$

$$= \frac{3\sqrt{3}\,x^2}{2\pi} - \frac{3x^2}{4\pi}\Big|_0^{\pi/6} + \sin(x) - \frac{3x^2}{4\pi}\Big|_{\pi/6}^{\pi/3}$$

$$= \frac{3\sqrt{3}\,\pi^{\cancel{2}}}{72\cancel{\pi}} - \cancel{\frac{3\pi^2}{144\pi}} + \frac{\sqrt{3}}{2} - \frac{3\pi^{\cancel{2}}}{36\cancel{\pi}} - \frac{1}{2} + \frac{3\pi^2}{\cancel{144\pi}}$$

$$= \frac{\pi\sqrt{3}}{24} + \frac{\sqrt{3}}{2} - \frac{\pi}{12} - \frac{1}{2}$$

2

$$= \boxed{\frac{(\sqrt{3}-2)\pi}{24} + \frac{\sqrt{3}-1}{2}} \qquad \left(= \frac{(\sqrt{3}-2)\pi + 12(\sqrt{3}-1)}{24} \right)$$

(Be very strict here, no points w/o full details; answer is in back of book in this form)

6.3/12 $\quad V = \int_0^{\pi/2} \pi \left(\sqrt{\cos x \sin x}\right)^2 dx$ 1/

$$= \pi \int_0^{\pi/2} \cos x \sin x \, dx \qquad \text{Let } u = \sin x, \ du = \cos x \quad 1/$$

$$= \pi \int_{u=0}^{u=1} u \, du$$

$$= \pi \frac{u^2}{2}\Big|_0^1 = \boxed{\frac{\pi}{2}}$$ 1/ 1/

6.3/58 Note the solid is obtained by rotating the region under the "upper branch" of the hyperbola and above the x-axis, so bounded by $y = \sqrt{1+x^2}$, $x=-a$, $x=a$

$$V = \int_{-a}^{a} \pi \left(\sqrt{1+x^2}\right)^2 dx \quad 2/$$

$$= \pi \int_{-a}^{a} 1 + x^2 \, dx$$

$$= \pi \left[x + \frac{x^3}{3}\right]\Bigg|_{-a}^{a} \quad 1/$$

$$= \boxed{2\pi\left(a + \frac{a^3}{3}\right)} \quad 1/$$