4, 4, 2, 2, 4 = 16 total

MATH 134 – Problem Set 2, part B

Section 5.5

(4) 44. (a) – (i)
(b) – (ii)
(c) – (iv)
(d) – (iii)

(4) 45. (a) A(x) is increasing on (0,4) and (8,12)
(b) A(x) has a local maximum at $x=4$
a local minimum at $x=8$
(c) A(x) has inflection points at roughly $x = 1.5, 5.8, 10$
(where $f'(x)=0$. Note $A''(x) = \frac{d}{dx}\left(\frac{d}{dx}\int_0^x f(t)\,dt\right)$

$= \frac{d}{dx} f(x) = f'(x)$)

(d) A(x) concave up roughly (0, 1.5), (5.8, 10)
down (1.5, 5.8) (10, 12.5)

Section 5.4

(2) 10. $\int_{-2}^{2} 10x^9 + 3x^5\,dx = x^{10} + \frac{1}{2}x^6 \Big|_{-2}^{2} = \boxed{0}$

(2) 12. $\int_{-1}^{1} 5u^4 + u^2 - u\,du = u^5 + \frac{1}{3}u^3 - \frac{1}{2}u^2 \Big|_{-1}^{1} = (1 + \frac{1}{3} - \frac{1}{2}) - (-1 - \frac{1}{3} - \frac{1}{2})$

$= 2 + \frac{2}{3} = \boxed{\frac{8}{3}}$

62. The area of the triangle is $\frac{1}{2}$ (base)(height)

$A_{tri} = \frac{1}{2}(b-a)\cdot\left(\frac{a+b}{2} - a\right)\left(-\frac{a+b}{2} + b\right)$

$$= \frac{1}{2}(b-a)\cdot\left(\frac{b-a}{2}\right)\left(\frac{-a+b}{2}\right)$$

$$= \frac{(b-a)^3}{8} \quad 1$$

The area under the parabola is

$$A_{par} = \int_a^b (x-a)(b-x)\,dx$$

$$= \int_a^b -x^2 + (a+b)x - ab\,dx$$

$$= -\frac{x^3}{3} + \frac{a+b}{2}x^2 - abx \Big|_a^b$$

$$= \frac{-b^3}{3} + \frac{(a+b)b^2}{2} - ab^2 - \left(-\frac{a^3}{3} + \frac{(a+b)a^2}{2} - a^2b\right)$$

$$= -\frac{b^3}{3} + \frac{ab^2}{2} + \frac{b^3}{2} - ab^2 + \frac{a^3}{3} - \frac{a^3}{2} - \frac{a^2b}{2} + a^2b$$

$$= \frac{b^3}{6} - \frac{1}{2}b^2a + \frac{1}{2}ba^2 - \frac{1}{6}a^3$$

$$= \frac{1}{6}(b-a)^3 \quad 2$$

Now $\frac{4}{3}A_{tri} = \frac{4}{3}\cdot\frac{(b-a)^3}{8} = \frac{1}{6}(b-a)^3 = A_{par}$

So the area of the parabolic arch is $\frac{4}{3}$ of the area of the triangle.