MATH 134 PS 1 B Solutions

Section 5.1

26. the terms are $\sqrt{j+j^3}$ for $j=1,\dots,n$, so this is $\boxed{\sum_{j=1}^{n} \sqrt{j+j^3}}$

27. the terms are $\frac{j}{(j+1)(j+2)}$ for $j=1,\dots,n$, so the sum is $\boxed{\sum_{j=1}^{n} \frac{j}{(j+1)(j+2)}}$

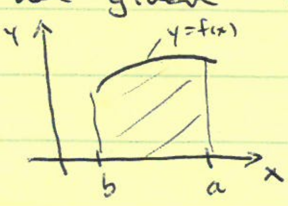
64. $\sum_{j=1}^{N} \left(2+\frac{3j}{N}\right)^4 \cdot \frac{3}{N}$ is the R_N sum for $f(x)=x^4$ and the subdivision of $[2,5]$ into N equal subintervals. So

$$\lim_{N\to\infty} \sum_{j=1}^{N} \left(2+\frac{3j}{N}\right)^4 \cdot \frac{3}{4} = \int_2^5 x^4\,dx$$

which gives the area between the x-axis and $y=x^4$ for $2 \le x \le 5$.

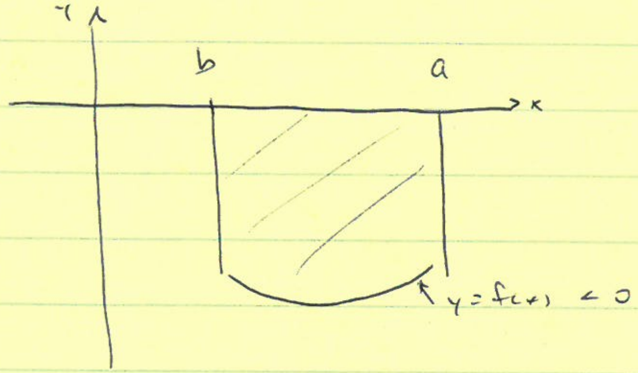
Section 5.2

82. (a) **False**: we're not given $a < b$, so if the graph was



$\int_a^b f(x)\,dx = -\int_b^a f(x) < 0.$

(b) Similarly, this statement is false as well. Here's an example

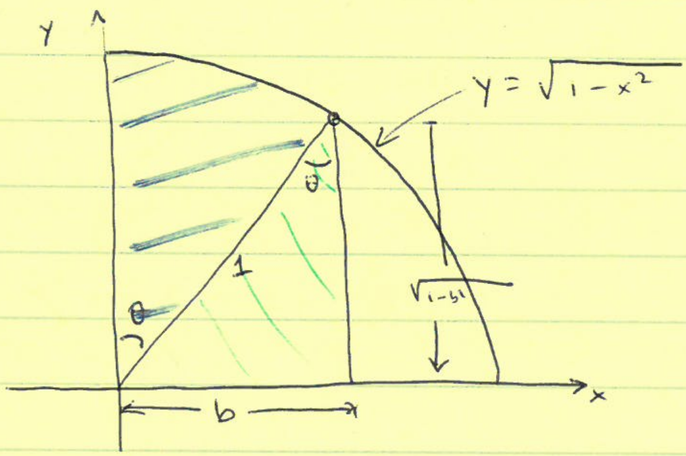


$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$= -(-\text{area}) > 0,$$

but $f(x) < 0$ on $[b, a]$.

5.2/86 (Extra Credit)



(unit circle, top half)

$$\int_0^b \sqrt{1-x^2}\,dx = \text{total area } (// + \backslash\backslash)$$

$$= \text{triangle area} + \text{sector area}$$

$$= \frac{1}{2} b\sqrt{1-b^2} + \frac{1}{2}\cdot 1^2 \cdot \sin^{-1}(b)$$

$$= \frac{1}{2} b\sqrt{1-b^2} + \frac{1}{2}\sin^{-1}(b)$$

(the angle θ for the sector is the same as θ in the triangle (interior angles to a transversal line to two parallel lines). In the triangle $\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{b}{1} = b$, so $\boxed{\theta = \sin^{-1}(b)}$)