Mathematics 134 – Calculus 2 With Fundamentals
Exam 4 – Review Sheet
April 26, 2018

General Information

As announced in the course syllabus, the fourth midterm exam of the semester will be given in class on Friday, May 4. The format will be similar to that of the first three midterms.

- You may use a calculator. Graphing calculators are also OK, although I don't think you will need or want any graphing features for the material on this exam.
- Use of phones, I-pods, I-pads, and any electronic devices other than your calculator *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the last exam (Problem Sets 7, and 8). However some calculations may require you to use integration methods covered on Exams 2 and 3 (review integration by u-substitution, and integration by parts in particular.) The new material is:

- Improper integrals (section 7.7): Know how to recognize when an integral is improper, and how to evaluate these using limits. (Note: This was covered on the last exam too, but I'm giving everyone another chance here!) Note: L'Hopital's Rule might be needed to evaluate some limits involved here.
- Probability applications (section 7.8): Know how to recognize when a function is a valid probability density (pdf) or compute an undetermined constant to make a function a valid pdf. Know how to compute the probability a random variable has a value in a given interval given the pdf; know how to compute the expected value given the pdf.
- Know how to compute normal probabilities using a table like the one we discussed in class. Note: If you have used tables arranged according to a different format in another class and are more comfortable with them, you may bring a *clean, unmarked* copy of your table with you and use it on the test. However I will inspect the sheets you bring to make sure there is no other "unauthorized" information recorded on them. *If I do find such information, I will discard the sheet and give you a copy of our table.*
- Differential equations, separation of variables, solutions, etc. (section 9.1)
- Models involving $y' = k(y - b)$ (section 9.2): Newton's Law of Heating/Cooling, etc. "word problems" based on that.
- Section 9.3: The material on slope fields we discussed briefly in class *will not be covered* on this exam or the final.

- Logistic equations (section 9.4): Know how to derive the general solution of $y' = ky(1 - y/A)$ and how to use those solutions to solve initial value problems.

- Time value of money concepts: compound interest (compounding monthly, continuously), present and future value of an asset. Future value of an "annuity" (i.e. a steam of equal payments at regular intervals for a fixed number of intervals). Determining the monthly payment on a loan, given the principal (amount borrowed), the interest rate, and the number of years to repayment. A copy of the formula sheet from class on Thursday, April 26 will be provided.

Sample problems

Note: The actual exam will be considerably shorter than the following list of questions and it might not contain questions of all of these types. The purpose here is just to give an idea of the range of different topics that will be covered and how questions might be posed.

I. For each of the following integrals, say why the integral is improper, determine if the integral converges, and if so, find its value.

A) $\displaystyle\int_1^\infty \frac{1}{\sqrt[5]{x}}\,dx$

B) $\displaystyle\int_0^2 \frac{dx}{x^2 - 7x + 6}$

C) $\int_0^\infty xe^{-3x}\,dx$

D) (More Challenging) For which values of a is $\int_0^\infty e^{ax}\sin(x)\,dx$ convergent? Evaluate the integral for those a.

II. (Probability) The time t (in minutes) between two successive incoming calls at a phone center is a random variable with a pdf of the form $f(t) = ce^{-t/r}$ if $t \geq 0$ and 0 otherwise.

(A) What must the value of c be to make this a valid pdf? Express c in terms of the constant r.

(B) Suppose that the probability that two successive calls are more than one minute apart is .3. What is the value of r?

(C) Using the value of r you determined in part A, find the probability that the two successive calls are at least 5 minutes apart.

III. (Normal probabilities) Using the normal curve area table to answer these questions.

(A) What is the probability that a standard normal random variable Z satisfies $-1.30 < Z < 0.83$?

(B) Cans of tomato soup have a stated volume of 16 fluid ounces, but the actual volume of the contents is normally distributed with mean 16.3 fluid oz. and SD .22 fluid oz. What is the probability that a randomly selected can contains less than 16 fluid oz. of tomato soup?

IV. *Differential Equations.*

(A) Verify that for any constant C, the function $y = \sqrt{C + x^2}$ is a solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x}{y}$. Which of these solutions also satisfies the initial condition $y(0) = 5$?

(B) For which real a is $y = e^{ax}$ a solution of the differential equation $\frac{dy}{dx} - 16y = 0$?

(C) Find the general solution of

$$\frac{dy}{dx} = xe^{x+y}$$

(D) (More Challenging) Find the general solution of

$$\sqrt{1 - x^2}\frac{dy}{dx} = y^2 + 2y$$

(E) A baked potato is taken out of the oven at a temperature of 140° C and left to cool on a counter in a room maintained at 20° C. After 2 minutes the temperature of the potato has decreased to 100° C. How long will it take for the temperature to reach 80° C?

(F) Sunset Lake is stocked with 2000 rainbow trout. After 1 year the trout population has grown to 4500. Assuming logistic growth with a carrying capacity of 10000 trout, find the growth constant k and determine when the population will reach 5000.

V. Time value of money (and geometric series).

(A) Juan deposits \$500 in a savings account earning 3% interest compounded monthly. What will the balance in the account be after 4 years?

(B) What is the sum of the finite geometric series

$$6 + \frac{6}{4} + \frac{6}{16} + \cdots + \frac{6}{4^9}?$$

What happens when you keep adding terms of this form indefinitely:

$$6 + \frac{6}{4} + \frac{6}{16} + \cdots + \frac{6}{4^{N-1}}$$

and take a limit as $N \to \infty$?

(C) Some banks began offering "Christmas clubs" around the time of the Great Depression in the 1930's. Customers would make contributions from their weekly pay checks to the "club" in order to have a given amount on hand to buy presents come Christmas time. Suppose you decide to make 48 weekly payments into a Christmas club earning an annual interest rate of 1%, compounded weekly. What should the payment be in order to have \$800 on hand to spend on presents?

(D) What would be the monthly payment on a 15-year mortgage for $300,000$ at an annual interest rate of 3.6%?