MATH 134 – Calculus with Fundamentals 2
Discussion Day on Trigonometric Substitution Integrals
March 21, 2018

Background

Recall that we saw some first examples of the *trigonometric substitution* method last time. The basic outline of this method is that for integrals involving

- $\sqrt{a^2 - x^2}$, we let $x = a\sin\theta$ and $dx = a\cos\theta\ d\theta$
- $\sqrt{a^2 + x^2}$, we let $x = a\tan\theta$ and $dx = a\sec^2\theta\ d\theta$
- $\sqrt{x^2 - a^2}$, we let $x = a\sec\theta$ and $dx = a\sec\theta\tan\theta\ d\theta$

(The last one is only included for completeness and possible future reference. We will only be doing integrals involving the sin and tan subsitutions.) We simplify, then apply our trig reduction formulas from last week. The last step is to convert back to functions of x using the reference triangle corresponding to the substition used:

- For the $x = a\sin\theta$ substitution, put x on opposite side, a on hypotenuse, then $(adj) = \sqrt{a^2 - x^2}$, so you can read off any trig function of θ from the triangle and $\theta = \sin^{-1}(x/a)$
- For the $x = a\tan\theta$ substitution, put x on opposite side, a on adjacent, then $(hyp) = \sqrt{x^2 + a^2}$, so you can read off any trig function of θ from the triangle and $\theta = \tan^{-1}(x/a)$
- For the $x = a\sec\theta$ substitution, put x on hypotenuse, a on adjacent, then $(opp) = \sqrt{x^2 - a^2}$, so you can read off any trig function of θ from the triangle and $\theta = \sec^{-1}(x/a)$. (This case is included for completeness, you are only responsible for the first two.)

Questions

(A) Using the appropriate trigonometric substitution compute

$$\int \frac{1}{\sqrt{x^2 + 25}}\ dx$$

(B) Using the appropriate trigonometric substitution compute

$$\int \frac{1}{(36 - x^2)^{3/2}}\, dx$$

(C) You want to divide a circular pizza with radius 9in (say with outer crust along the circle $x^2 + y^2 = 81$) into three exactly equal pieces with a knife or pizza cutter. One way, of course, is to divide it into three sectors each with angle exactly $2\pi/3$ by cutting from the center to the outer edge. But another (easier?) way would be to cut the pizza with two parallel strokes, say at $x = \pm a$ so that the three strips $-9 \leq x \leq -a$, $-a \leq x \leq a$, and $a \leq x \leq 9$ all have the same area. Set up an integral with a in the limits of integration, evaluate it using an appropriate trigonometric substitution, and find an equation to solve for the a giving three strips of exactly equal area. Find an approximation for a by using a graphing calculator.