MATH 134 – Calculus with Fundamentals 2
Discussion Day 2 – More on volumes with "washer" cross-sections
February 22, 2018

Background

Recall that when a solid of revolution has "washer" cross sections with outer radius r_{out} and inner radius r_{in} (both possibly depending on x), the volume is computed by

$$\text{Volume} = \int_a^b \pi(r_{out})^2 - \pi(r_{in})^2 \, dx$$

Questions

(1) Find the volume of the solid obtained by rotating the region bounded by $y = 4$, $y = x/2$ for $x = 0$ to $x = 1$ about the x-axis. (It's a solid cylinder with a cone "drilled out.") First draw the region, then identify r_{out}, r_{in} for the rotation about the x-axis, set up the volume integral, then carry out the integration.

(2) Same region as in (1), but rotated about the vertical line $x = -1$.

(3) Now consider the solid obtained by rotating the region inside the circle $x^2 + (y - 3)^2 = 1$ about the x-axis. Since that region does not extend all the way down to the x-axis, this solid will also have washer cross-sections.

(a) Solve the equation of the circle for y to get

$$y = 3 \pm \sqrt{1 - x^2}$$

One of the choices of the sign will give you the outer radius and one will give you the inner radius. Which is which?

(b) Set up the volume integral. Note that the limits of integration correspond to the x-values farthest to the left and to the right on the given circle. What are they?

(c) Square out the two terms in $\pi(r_{out})^2 - \pi(r_{in})^2$, and simplify. Explain why what you get is

$$\text{Volume} = 12\pi \int_{-1}^{1} \sqrt{1 - x^2} \, dx$$

(d) The integral in part (c) is one you can do just by recognizing it as an *area*. What is the value? (*DON'T* try to find an antiderivative; we don't know how to do that yet!)

(e) A famous theorem of the ancient Greek mathematician Pappus of Alexandria (ca. 290 - ca. 350 CE) says (in a particular case) that the volume of the solid obtained by rotating a circular disk about an axis that does not intersect the circle is equal to *the area of the circle, times the distance the center of the circle travels as it rotates around the line.* Check that your answer from part (d) agrees with this(!)