Mathematics 134 – Calculus 2 With Fundamentals
Exam 3 – Review Sheet
April 5, 2018

General Information

As announced in the course syllabus, the third midterm exam of the semester will be given Friday, April 13 (not a bad omen, I hope!). The format will be similar to that of the other midterms.

- You may use a calculator – graphing calculators are OK, although I doubt you'll find any use for the graphing features on this exam.
- Use of cell phones, I-pods, I-pads, tablets, and any other electronic device besides a basic calculator *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (and make sure cell phones are turned off).

What will be covered

The exam will cover the material since the last exam (Problem Sets 5, 6 and the material on improper integrals from last week), namely the following material from sections 7.2, 7.3, 7.5, 7.6, 7.7 of Rogawski/Adams:

1. Trigonometric integrals using the trig reduction formulas – *I will provide a copy of the summary sheet on the reduction formulas for you to use*
2. Integration by trigonometric substitution
3. Integration by partial fractions
4. Improper integrals (integrals of functions with discontinuities and integrals over infinite intervals)

Important Note: Many of the problems on this exam will require you to set up and compute an integral to find the quantity that is asked for. The methods of integration tested on the first two exams (i.e. basic rules, u-substitution, integration by parts) and the new methods on this exam might be required to evaluate the integral. In other words, this exam is *in effect a cumulative exam on the material on methods of integration.* Especially there were things you had not mastered on the first two exams, you will probably want to begin your review for this exam by going back and looking at the material from sections Chapter 5 in Rogawski/Adams.

There will be a review for the exam in class on Thursday, April 12.

Review Problems from the text

Chapter 7 review problems: 3 (except change the directions to say "using the trigonometric reduction formulas"), 5, 6, 7, 8, 10 (a more specific direction: multiply the top and the bottom by x) 14, 15, 16, 19, 23, 27, 34, 42, 43, 75, 77, 82, 83

Sample Exam Questions

Disclaimer: The actual exam questions may be organized differently. In particular, I may ask one question about the trigonometric substitution and/or partial fractions methods the way I did on the quizzes instead of asking you to work out an integral from start to finish. This list is also somewhat longer than the actual exam will be (to give you some more problems to practice on).

I. Compute each of the integrals below the appropriate method or combination of methods. You must show all work for full credit.

A) Use the trigonometric reduction formulas to find

$$\int \sin^3(x)\cos^4(x)\ dx$$

B) Use the trigonometric reduction formulas to find

$$\int \sec^5(x)\ dx$$

C)

$$\int \sqrt{25 - x^2}\ dx$$

D) What changes in part C if the integral is

$$\int \sqrt{25 + x^2}\ dx$$

Work this one out too, start to finish.

E)

$$\int \frac{x^3 + 2x + 1}{x^2 + 6x - 7}\ dx$$

F)

$$\int \frac{1}{x^2(x^2 + 6x + 10)}\ dx$$

II. (Improper integrals)

A) Why is the following integral an *improper integral.* Decide whether it converges by setting up and evaluating the appropriate limit(s):

$$\int_1^3 \frac{1}{(x-3)^{2/3}}\, dx$$

B) Follow-up to part A: For which exponents $\alpha > 0$ will the improper integral

$$\int_1^3 \frac{1}{(x-3)^{\alpha}}\, dx$$

converge?

C) Why is the following integral an improper integral? Decide whether it converges by setting up and evaluating the appropriate limit(s):

$$\int_0^\infty xe^{-x}\, dx.$$

III.

(A) Verify that $\int \csc\theta\, d\theta = \ln|\csc\theta - \cot\theta| + C$ by differentiating.

(B) Which trigonometric substitution would you apply to compute $\int \frac{1}{u\sqrt{a^2-u^2}}\, du$? What trigonometric integral do you get after making the substitution? Complete the computation of the integral.

(C) Our textbook's table of integrals gives this one as

$$\frac{-1}{a} \ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

Show that the form you got in part B is equivalent to this.