Mathematics 134 – Calculus With Fundamentals 2
Exam 1 – Review Sheet
February 9, 2018

*Sample Exam Questions- Solutions*

I.

(A) Since the interval is $[0, 1]$ and $n = 4$, the sums are

$$L_4 = f(0)(.25) + f(.25)(.25) + f(.5)(.25) + f(.75)(.25) = .96875$$

$$R_4 = f(.25)(.25) + f(.5)(.25) + f(.75)(.25) + f(1)(.25) = 1.71875$$

$$M_4 = f(.125)(.25) + f(.375)(.25) + f(.625)(.25) + f(.875)(.25) = 1.328125$$

(B) $f'(x) = 2x + 2 > 0$ for all $x \in [0, 1]$. Hence $f$ is *increasing* on this interval. This implies that the left-hand Riemann sum is less than $\int_0^1 x^2 + 2x\ dx$, and the right-hand Riemann sum is greater than the value of the integral. Using the Evaluation Theorem, we can check this:

$$\int_0^1 x^2 + 2x\ dx = \frac{x^3}{3} + x^2 \Big|_0^1 = \frac{4}{3} = 1.\overline{3}.$$

(C) The sum is the $R_N$ sum for $f(x) = \frac{\cos(x)}{1+x^2}$ on $[0, \pi]$, so the limit is

$$\int_0^\pi \frac{\cos(x)}{1 + x^2}\ dx.$$

II. Let

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 3 \\ x - 2 & \text{if } 3 \le x \le 5 \\ 13 - 2x & \text{if } 5 \le x \le 8 \end{cases}$$

(A) Sketch the graph $y = f(x)$. The graph is made up of segments of three different straight lines. See figure at top of next page.

In the rest of the parts, $F(x) = \int_0^x f(t)\ dt$, where $f$ is the function from part A.

(B) Assuming $F(0) = 0$, Compute $F(1), F(2), F(3), F(4), F(5), F(6), F(7), F(8)$ given the information in the graph of $f$.

Using the area interpretation of the definite integral we have

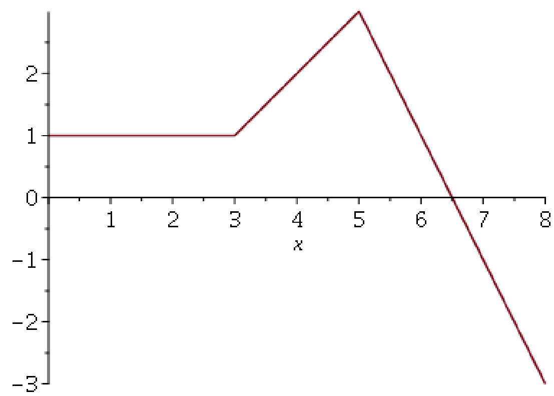


Figure 1: Figure for Problem II.

$$F(1) = \int_0^1 f(x)\,dx = 1$$

$$F(2) = \int_0^2 f(x)\,dx = 2$$

$$F(3) = \int_0^3 f(x)\,dx = 3$$

$$F(4) = \int_0^3 f(x)\,dx + \int_3^4 f(x)\,dx = 3 + \frac{3}{2} = \frac{9}{2}$$

$$F(5) = \int_0^4 f(x)\,dx + \int_4^5 f(x)\,dx = \frac{9}{2} + \frac{5}{2} = 7$$

$$F(6) = \int_0^5 f(x)\,dx + \int_5^6 f(x)\,dx = 7 + 2 = 9$$

$$F(7) = \int_0^6 f(x)\,dx + \int_6^{13/2} f(x)\,dx + \int_{13/2}^7 f(x)\;dx = 9 + \frac{1}{4} - \frac{1}{4} = 9$$

$F(8) = \int_0^5 f(x)\,dx + \int_5^{13/2} f(x)\,dx + \int_{13/2}^8 f(x)\,dx = \int_0^5 f(x)\,dx = 7$ (the last two integrals cancel since they represent equal areas with opposite signs).

(C) By the Fundamental Theorem of Calculus, $F'(x) = f(x)$. Since $f(13/2) = 0$, the point

$x = 13/2$ is a critical point. Since $F' = f$ changes sign from positive to negative at the critical point, $x = 13/2$ is a local maximum.

(D) Since $G(x) = \int_2^x f(t)\ dt = \int_0^x f(t)\ dt - \int_0^2 f(t)\ dt = F(x) - 2$, the graph $y = G(x)$ is obtained from $y = F(x)$ by shifting down 2 units along the $y$-axis.

III. Find the derivatives of the following functions

(A) $f(x) = \int_0^x \sin(t)/t\, dt$.

$$f'(x) = \frac{\sin x}{x}$$

(B) $g(x) = \int_5^{x^3} \tan^4(t)\ dt$.

$g(x) = m(x^3)$, where $m(x) = \displaystyle\int_5^x \tan^4(t)\ dt$. Then, $g'(x) = m'(x^3) \cdot 3x^2 = \tan^4(x^3) \cdot 3x^2$.

(C)) $h(x) = \int_{-3x}^{5x} e^{t^2} \sin(t)\ dt$.

$h(x) = n(x) + l(x)$, where $n(x) = \displaystyle\int_{-3x}^{0} e^{t^2} \sin(t)\ dt$ and $l(x) = \displaystyle\int_0^{5x} e^{t^2} \sin(t)\ dt$. Then $h'(x) = n'(x) + l'(x) = -(e^{(-3x)^2} \sin(-3x)) \cdot (-3) + 5 \cdot e^{(5x)^2} \sin(5x) = 3e^{9x^2} \sin(-3x) + 5e^{25x^2} \sin(5x)$.

IV.

(A) Compute $\int 5x^4 - 3\sqrt{x} + e^x + \frac{2}{x}\ dx$

$$\int 5x^4 - 3\sqrt{x} + e^x + \frac{2}{x}\ dx = x^5 - 2x^{3/2} + e^x + 2\ln|x| + C$$

(B) Apply a $u$-substitution to compute $\int x(4x^2 - 3)^{3/5}\ dx$

$u = 4x^2 - 3$, $du = 8x\, dx$. Then $\displaystyle\int x(4x^2 - 3)^{3/5}\ dx = \int \frac{1}{8} u^{3/5}\, dx = \frac{1}{8}\frac{u^{8/5}}{8/5} + C = \frac{5}{64}(4x^2 - 3)^{8/5} + C$

(C) Apply a $u$-substitution to compute $\int_1^2 e^{\sin(\pi x)} \cos(\pi x)\ dx$

$u = \sin(\pi x)$, $du = \pi \cos(\pi x)$. Then $\displaystyle\int_1^2 e^{\sin(\pi x)} \cos(\pi x)\ dx = \frac{1}{\pi} \int_0^0 e^u\, du = 0$

(D) Let $u = t^3 + 3t + 3$, then $du = (3t^2 + 3)\ dt = 3(t^2 + 1)\ dt$. Then given integral is

$$\int \frac{t^2 + 1}{t^3 + 3t + 3}\ dt = \int \frac{1}{3u}\ du = \frac{1}{3} \ln|u| + C = \frac{1}{3} \ln|t^3 + 3t + 3| + C.$$

(E) Let $u = -7x$ or "guess and check." *Answer*: $\frac{-1}{7}e^{-7x} + C$.
(F) One solution: Let $u = \sin(2x)$. Then $du = 2\cos(2x)\ dx$, so the given integral is

$$\int u \cdot \frac{1}{2}\ du = \frac{1}{4}u^2 + C = \frac{1}{4}\sin^2(2x) + C.$$

You can equally well let $u = \cos(2x)$ and then $du = -2\sin(2x)\ dx$ and the given integral is

$$\int u \cdot \frac{-1}{2}\ du = -\frac{1}{4}u^2 + C = -\frac{1}{4}\cos^2(2x) + C.$$

Both answers are correct, and each differs from the other by an additive constant because of the identity $\cos^2(2x) + \sin^2(2x) = 1$ for all $x$.