Mathematics 134 – Calculus With Fundamentals 2
Exam 1 – Review Sheet
February 9, 2018

General Information

As announced in the course syllabus, the first midterm exam of the semester will be given in class on Friday, February 16. There will be four or five questions (each one with several parts), similar in format to the sample exam questions below.

- You may use a scientific calculator for the exam (graphing calculators OK).
- Use of phones, tablets, I-pods, and all other electronic devices besides a basic calculator *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the start of the semester – Problem Sets 1, 2, including the following material from sections 5.1 through 5.5, 5.7 of Rogawski and Adams. Note that there *will be* a u-substitution problem on this exam, even though that topic was not covered in the first two problem sets. See practice questions below for the type of question I might ask.

1) Riemann sums and the definition of the definite integral.
2) The Fundamental Theorem of Calculus: Part II: If $f(t)$ is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)\, dt$, then $F'(x) = f(x)$. Part I: (the "Evaluation Theorem") If $G(x)$ is an antiderivative of a continuous function $f(x)$ on $[a, b]$, then

$$\int_a^b f(x)\, dx = G(b) - G(a)$$

   Recall, though: our discussion of these topics in class was slightly different from the way they are presented in the text. I will not ask you about the way Rogawski and Adams prove the "Evaluation Theorem" in their Section 5.4, but I might ask you about the technique we used to prove the second part of the Fundamental Theorem (see p. 295-296).
3) Antiderivatives (indefinite integrals) and basic antiderivative rules: All rules coming from basic derivative formulas: Know $\int x^n\, dx$, $\int e^x\, dx$, $\int \sin(x)\, dx$, $\int \cos(x)\, dx$, $\int \frac{1}{x^2+1}\, dx$, $\int \frac{1}{\sqrt{1-x^2}}\, dx$, and so forth, plus the sum, and constant multiple rules
4) Integrals by u-substitution

There will be a review for the exam in class on Thursday, February 15. Tori will also be doing her usual evening session that night.

Review Problems

The Review Problems 1–30, and 39–118 at the end of Chapter 5 are good for preparation for this exam. It's not necessary to work all of them. But you should try a good selection and practice choosing a method at least for most of the integrals in problems 17–30 and 43–75.

Sample Exam Questions

Note: The actual exam will be definitely shorter than the following list of questions. The purpose here is just to give an idea of the range of different topics that will be covered and how questions might be posed.

I.

A) Evaluate the L_4, R_4, and M_4 Riemann sums for $f(x) = x^2 + 2x$ for $0 \le x \le 1$ (Note: you are using $N = 4$ each time).

B) In part A), one of your values is definitely larger than the actual value of $\int_0^1 x^2 + 2x\,dx$ and one is definitely smaller than the integral. Which is which? (Answer without calculating the value of the integral and then check your work.)

C) The following limit represents a definite integral:

$$\lim_{N\to\infty} \sum_{j=1}^{N} \frac{\cos\left(\frac{j\pi}{N}\right)}{1 + \left(\frac{j\pi}{N}\right)^2} \frac{\pi}{N}$$

What is the integral? (Note: This is just asking for the definite integral, not for the number that integral would compute.)

II. Let

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 3 \\ x - 2 & \text{if } 3 \le x \le 5 \\ 13 - 2x & \text{if } 5 \le x \le 8 \end{cases}$$

A) Sketch the graph $y = f(x)$. In the rest of the parts, $F(x) = \int_0^x f(t)\,dt$, where f is the function from part A.

B) Compute $F(1), F(2), F(3), F(4), F(5), F(6), F(7), F(8)$ given the information in the graph of f.

C) Are there any critical points of F (places where $F'(x) = 0$)? If so, find them and say whether they are local maxima, local minima, or neither. If not, say why not.

D) How is the graph of $F(x)$ related to the graph of

$$G(x) = \int_2^x f(t)\,dt?$$

III. Find the *derivatives* of the following functions:

A) $f(x) = \int_0^x \sin(t)/t\,dt$.

B) $g(x) = \int_5^{x^3} \tan^4(t)\ dt$.
C) $h(x) = \int_{-3x}^{5x} e^{t^2} \sin(t)\ dt$.

IV.

A) Compute $\int 5x^4 - 3\sqrt{x} + e^x + \frac{2}{x}\ dx$

B) Apply a u-substitution to compute $\int x(4x^2 - 3)^{3/5}\ dx$

C) Apply a u-substitution to compute $\int_1^2 e^{\sin(\pi x)} \cos(\pi x)\ dx$

D) Compute

$$\int_0^1 \frac{t^2 + 1}{t^3 + 3t + 3}\ dt$$

(Hint: How does the bottom relate to the top?)
E) Compute $\int e^{-7x}\ dx$.
F) $\int \cos(2x) \sin(2x)\ dx$