MATH 133 – Calculus with Fundamentals 1
Problem Set 6B Solutions, November 3, 2017

Section 3.3

49. Since $f(x) = x^2 e^{-x}$, we can rewrite this as $f(x) = \frac{x^2}{e^x}$ and apply the quotient rule to get

$$f'(x) = \frac{e^x \cdot 2x - x^2 e^x}{(e^x)^2} = \frac{x(2-x)}{e^x}$$

after simplifying. We are looking for $x = a$ where the tangent line passes through the origin. That is clearly true if $a = 0$, but that is not the one we are looking for – there is also one with $a > 0$. The tangent line at $x = a$ is

$$y - \frac{a^2}{e^a} = \frac{a(2-a)}{e^a}(x-a)$$

This goes through the origin if the equation obtained by setting $x = 0$ and $y = 0$ in the equation of the tangent line is satisfied:

$$-\frac{a^2}{e^a} = \frac{a^3 - 2a^2}{e^a}$$

This implies $-a^2 = a^3 - 2a^2$, so

$$a^3 - a^2 = a^2(a-1) = 0.$$

That shows $a = 1$ is the other solution.

Section 3.4

2. The volume is $v(s) = s^3$. The rate of change of the volume with respect to s is the *derivative* $v'(s) = 3s^2$. When $s = 5$, $v'(5) = 3 \cdot 5^2 = 75$. (The units are not given, but if s was measured in meters, say, then the units of the derivative would be cubic meters per meter.)

38. The height looks like a sinusoid of the form $y = A\sin(Bt)$ for some constants A, B. The velocity is the derivative of height with respect to t. Its graph should look like a cosine $y = AB\cos(Bt)$. Since no units are given on the axes, any graph that looks like a cosine graph (maximum at 0, decreasing to zero at one-quarter of the way along, and reaching a minimum at the half-way point, then increasing back up to zero at three-quarters, and back up to the maximum at the end of the interval shown) is OK.

Section 3.5

37.

(a) $s(t) = 300t - 4t^3$, so $v(t) = s'(t) = 300 - 12t^2$, and $a(t) = v'(t) = s''(t) = -24t$. At $t = 5$, $a(5) = -24 \cdot 5 = -120$ (units are meters per minute per minute = meters per minute squared).

(b) The graph is a straight line going through the points $(t, a) = (0, 0)$ and $(t, a) = (6, -144)$. $a(t) = v'(t) < 0$ on the whole interval so $v(t)$ is decreasing. That says the helicopter is slowing down the whole time.

40. One of the two graphs (A) and (B) must show the derivative of the other. The function in graph (B) would have derivative equal to zero at the tops of the hills and the bottoms of the valleys – that's three x-values in all, including $x = 0$, and two other $x > 0$. That matches the graph in (A). So (B) is $y = f(x)$ and (A) is $y = f'(x)$. Then (C) is $y = f''(x)$.