MATH 133 – Calculus with Fundamentals 1
Problem Set 1B Solutions, September 8, 2017

1.1/71. Given: The function $f(x)$ has domain $[4, 8]$ and range $[2, 6]$.

(a) The function $f(x)+3$ has domain $[4, 8]$ (the same as $f(x)$) and range $[5, 9]$, since $2 \leq f(x) \leq 6$ is equivalent to $5 \leq f(x) + 3 \leq 9$.

(b) The function $f(x + 3)$ has domain $[1, 5]$ since $4 \leq x + 3 \leq 8$ is the same as $1 \leq x \leq 5$. The range is the same as the range of $f(x)$ – $[2, 6]$.

(c) The function $f(3x)$ has domain $[4/3, 8/3]$ since $4 \leq 3x \leq 8$ is equivalent to $4/3 \leq x \leq 8/3$. The range is $[2, 6]$, the same as for $f(x)$.

(d) The function $3f(x)$ has domain $[4, 8]$, the same as $f(x)$. The range is $[6, 18]$, since $2 \leq f(x) \leq 6$ is equivalent to $6 \leq 3f(x) \leq 6$.

77. Another way to state the definition is

$$f(x) = \begin{cases} x & \text{if } x \geq 2 - x \\ 2 - x & \text{if } 2 - x > x. \end{cases}$$
$$= \begin{cases} x & \text{if } x \geq 1 \\ 2 - x & \text{if } x < 1. \end{cases}$$

This shows the graph has the form given in Figure 1 on the back of this page. From this, we see that this is the graph $y = |x|$, shifted to the right one unit and up one unit. Another formula is

$$y = |x - 1| + 1.$$

1.2/22. The line contains the points $(0, b)$ and $(a, 0)$. This means the slope is

$$m = \frac{b - 0}{0 - a} = -\frac{b}{a}.$$

The point $(0, b)$ is the y-intercept, so the equation of the line has the slope-intercept form:

$$y = -\frac{b}{a}x + b.$$

From this, we can add $\frac{b}{a}x$ to both sides to obtain

$$\frac{b}{a}x + y = b,$$

then divide both sides by b to obtain

$$\frac{x}{a} + \frac{y}{b} = 1.$$

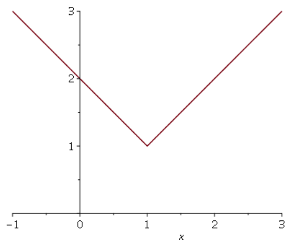

Figure 1: The graph $y = f(x)$ in 1.1/77

This is the form we wanted.

1.2/48. Saying $f(x) = Ax^2 + Bx + C$ has a *double* root means that its factored form is $A(x-r)^2$ for some r (equivalently, if you use the quadratic formula to solve $Ax^2 + Bx + C = 0$, then the $B^2 - 4AC$ under the radical sign is *zero* and you only get one root). Graphically, that means the parabola $y = Ax^2 + Bx + C$ is "just touching" the x-axis at $x = r$. (Graph $y = (x-2)^2$ to see exactly what that looks like.) It is statement (a) that is correct, since any parabola can be *shifted up or down* by a unique constant c to make it just touch the x-axis at one point. (b) cannot be true because shifting the parabola left or right does not change the number of times it crosses the x-axis; it just changes the *locations of the points* (if there are any) where the parabola crosses the x-axis.