College of the Holy Cross
MATH 133, Calculus With Fundamentals 1
Final Examination – Friday, December 15

Your Name: ______________________________

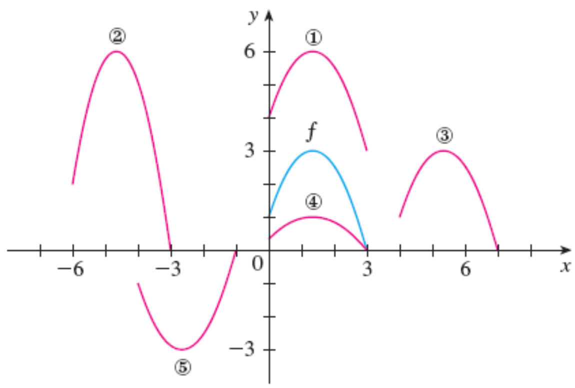
Instructions: For full credit, you must show *all work* on the test pages. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values. There are 200 total points on this exam.

Please do not write in the space below

Problem	Points/Poss
I	/ 25
II	/ 25
III	/ 40
IV	/ 45
V	/ 25
VI	/ 20
VII	/ 20
Total	/ 200

HAPPY HOLIDAYS!!!

I. The graph $y = f(x)$ is given in light blue. Match each equation with one of the numbered magenta graphs.



(5) A) $y = 2f(x+6)$ is plot number: ________

(5) B) $y = \frac{1}{3}f(x)$ is plot number: ________

(5) C) $y = f(x) + 3$ is plot number: ________

(5) D) $y = f(x-4)$ is plot number: ________

(5) E) $y = -f(x+4)$ is plot number: ________

II. A cup of hot chocolate is set out on a counter at $t = 0$. The temperature of the chocolate t minutes later is $C(t) = 65 + 55e^{-t/4}$ (in degrees F).

A) (5) What is the temperature of the chocolate at $t = 0$?

B) (10) What is the (instantaneous) rate of change of the temperature at $t = 10$ minutes?

C) (10) How long does it take for the temperature to reach $100°F$?

III. Compute the following limits. Any legal method is OK.

(A) (10) $\lim_{x \to 3} \frac{x^2 + x - 12}{x^2 - 5x + 7}$

(B) (10) $\lim_{x \to 1} \frac{x - 1}{x^2 - 1}$

(C) (10) $\lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x}$

(D) (10) $\lim_{x \to 0} \frac{\tan(x)}{x}$

IV.

A) (10) *Using the limit definition*, and showing all necessary steps to justify your answer, compute $f'(x)$ for $f(x) = 5x^2 + x$.

IV. (continued) Using appropriate derivative rules, compute the derivatives of the following functions. *You do not need to simplify your answers.*

B) (5) $g(x) = 4x^3 + \sqrt{x} + \dfrac{2}{\sqrt[4]{x}} + e^2$

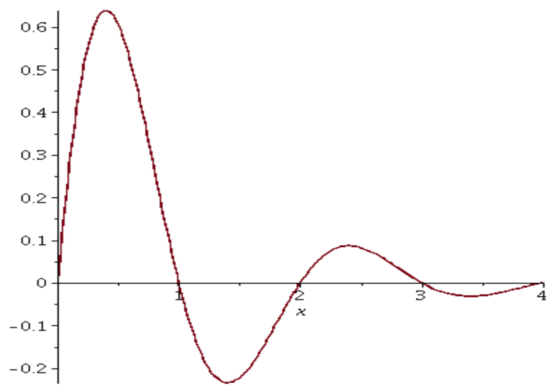
C) (10) $h(x) = \dfrac{\cos(x) + x}{\sin(x)}$

D) (10) $i(x) = \ln(x^3 + e^x)$

IV. (continued)

E) (10) $j(x) = \tan^{-1}(3x + 7)$

V. The following graph shows the *derivative* $f'(x)$ for some function $f(x)$ defined on $0 \leq x \leq 4$. Note: This *is not* $y = f(x)$, it is $y = f'(x)$.



Using the graph, *estimate*

A) (5) The interval(s) on which $f(x)$ is increasing.

Increasing on:

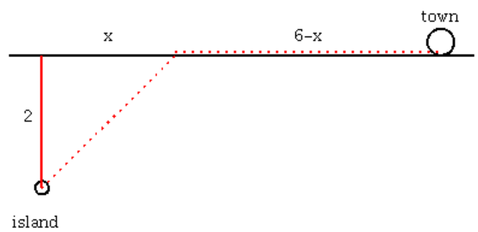
B) (10) The critical numbers of $f(x)$ *in the open interval* $(0, 4)$. Say what the behavior of $f(x)$ is at each critical number (local max, local min, neither).

Critical Points and behaviors:

C) (10) The interval(s) on which $y = f(x)$ is concave down.

Concave down on:

VI. A town wants to build a pipeline from a water station on a small island 2 miles from the shore of its water reservoir to the town. One possible route is shown dotted in red. The town is 6 miles along the shore from the point nearest the island. It costs \$3 million per mile to lay pipe under the water and \$2 million per mile to lay pipe along the shoreline.



A) (5) Give the cost $C(x)$ of the pipeline as a function of the location x as shown.

B) (5) Differentiate your $C(x)$ to find $C'(x)$.

C) (5) Find all critical points of $C(x)$.

D) (5) Where on the shoreline should the pipeline hit land to minimize the costs of construction? Say how you know this gives the minimum cost.

VII. (20) A block of dry ice (solid CO_2) is evaporating and losing volume at the rate of 10 cm^3/min. It has the shape of a cube at all times. How fast are the edges of cube shrinking when the block has volume 216 cm^3?