College of the Holy Cross
MATH 133, section 1 – Calculus with Fundamentals
Final Exam – December 15, 2015

Your Name: ______________________________

Instructions: Except on problems I and V, you must *show all work necessary to justify your answers* on the test pages. Place your final answer in the space provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values. There are 200 total points on this exam.

Please do not write in the space below

Problem	Points/Poss
I	/ 30
II	/ 20
III	/ 30
IV	/ 50
V	/ 30
VI	/ 20
VII	/ 15
VIII	/ 5
Total	/ 200

HAVE A PEACEFUL AND ENJOYABLE HOLIDAY SEASON!

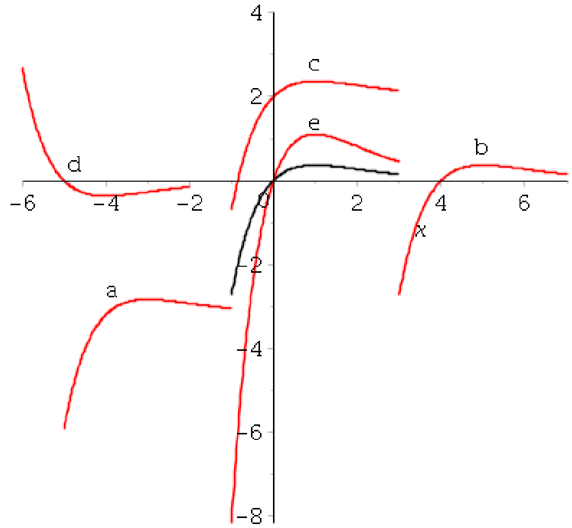

Figure 1: Figure for problem I

I. A portion of the graph $y = f(x)$ is given in black. Match each equation with one of the red graphs identified with lower-case letters.

A) (5) $y = -f(x+5)$ is letter: ________

B) (5) $y = f(x+4) - 3.2$ is letter: ________

C) (5) $y = 3f(x)$ is letter: ________

D) (5) $y = f(x) + 2$ is letter: ________

E) (5) $y = f(x-4)$ is letter: ________

F) (5) Is $f(x) = xe^{-x}$ ________ or $f(x) = \cos(x) - 1$ ________? (Mark the correct formula).

II. The power delivered by a battery to an apparatus of resistance R (in ohms) is

$$P(R) = \frac{5R}{(R+0.5)^2}$$

(in watts).

A) (5) If $R = 10$ ohms, what is the power delivered by the battery?

B) (5) A power of 2 watts can be obtained with two different values of the resistance R. What are they?

C) (10) What is the *rate of change* of the power at $R = 10$ ohms?

III. Compute the following limits. Any legal method is OK.

(A) (10) $\lim_{x \to 3} \dfrac{x^2 - 6x + 9}{x^2 - 5x + 6}$.

(B) (10) $\lim_{x \to 2} \dfrac{\sqrt{x+7} - 3}{x - 2}$.

(C) (10) $\lim_{x \to 0} x \cot(x)$

IV.

A) (10) *Using the limit definition*, and showing all necessary steps to justify your answer, compute $f'(x)$ for $f(x) = \dfrac{1}{x-3}$.

Using appropriate derivative rules, compute the derivatives of the following functions. You do *not* need to simplify your answers.

B) (10) $g(x) = \pi x^e + \dfrac{3}{x^7} + \sqrt[3]{x} + 5$

C) (10) $h(x) = \dfrac{x^2 - 4x + 1}{x^3 + 4}$

D) (10) $i(x) = x\ln(\cos(x)) + e^{\cos(x)}$

E) (10) $j(x) = \sin^{-1}(3x - 4)$

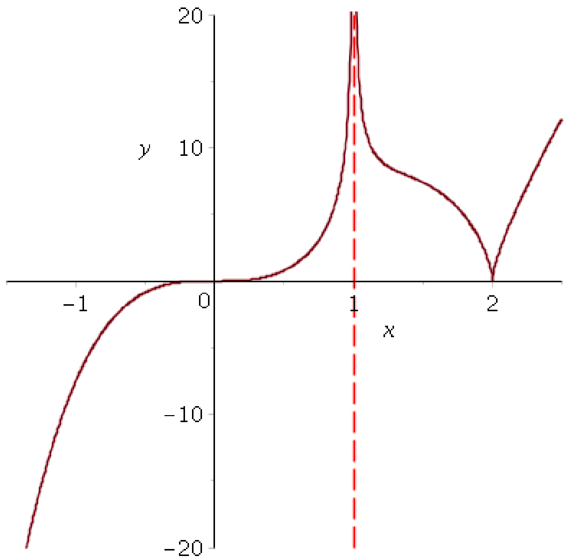

Figure 2: $y = f'(x)$ for problem V.

V. The graph in Figure 2 shows the *derivative* $f'(x)$ for some function $f(x)$ defined on $-1.5 \le x \le 2.5$. In particular you should assume $f(1), f(2)$ are defined and finite. Note: This graph *is not* $y = f(x)$, *it is* $y = f'(x)$. Using the graph, answer these questions:

A) (5) What are the critical points of f in the interval $[-1.5, 2.5]$?

Answer: $x =$ ____________________________

B) (5) Classify each of the points you found in part A) as a local maximum, a local minimum, or neither.

Answer: ____________________________

C) (5) Explain briefly how you know your answer in B) is correct.

D) (5) Is $x = 0$ a point of inflection of f? Why or why not?

Answer: ____

E) (5) Does it appear that $f''(2)$ exists? (Yes/No):

Answer: ____________

F) (5) Over which interval(s) contained in $[-1.5, 2.5]$ is the graph $y = f(x)$ concave down?

Answer: ____

VI. A Spanish factory can produce $P = 2LK^2$ million lightbulbs if labor costing L million euros is hired and equipment costing K million euros is obtained. If an order for 1.7 million lightbulbs is received, which combination of L and K will *minimize* the total cost of labor plus equipment to fill the order?

A) (5) Express the total cost of producing 1.7 million lightbulbs as a function of one of the two variables K, L. (Your choice, but choose wisely!)

B) (10) Find a critical point of your function from part A which is a realistic solution of this problem; solve for the other variable.

C) (5) How do you know that your solution *minimizes* the total cost?

VII. (15) A moth ball is evaporating and losing volume at the rate of .1 cm^3/week. It has the shape of a sphere at all times. How fast is the surface area of the moth ball shrinking when the radius of the ball is 1cm? (Note: A sphere of radius r has volume $V = \frac{4\pi r^3}{3}$ and surface area $A = 4\pi r^2$.)

VIII. (5) True/False: The graph obtained by shifting $y = \ln(x)$ vertically by 2 units can *also* be obtained from $y = \ln(x)$ by a horizontal compression. Explain your answer.