College of the Holy Cross, Fall Semester, 2017
MATH 133, Midterm 4
Thursday, December 7

Your Name: _____

Instructions: Please show all work necessary to justify your answers, and write your answers in the spaces provided. Use the back of the preceding page if you need more space for scratch work. There are 100 possible points distributed as below.

Please do not write in the space below

Problem	Points/Poss
1	/ 25
2	/ 20
3	/ 30
4	/ 25
Total	/100

1. Find $\frac{dy}{dx}$; do not simplify:

A) (7.5) $y = \frac{\ln(x)+x^2}{\cos(x)}$

B) (7.5) $y = \sin^{-1}(\sqrt{x})$

C) (10) $x^3y^2 - 4y + x^4 = 0$ (use implicit differentiation)

2. (20) A stationary observer watches a weather balloon being launched from a point 600 feet away from her position. The line of sight distance from the observer to the balloon is changing at a rate of 20 feet per second. How fast is the height of the balloon changing when the balloon is 400 feet above the ground?

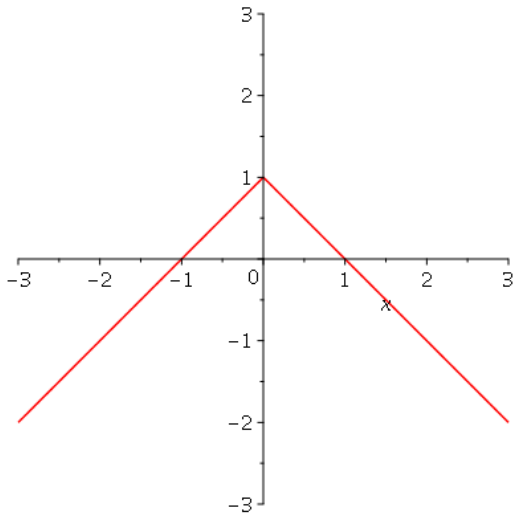

Figure 1: $y = f'(x)$ for Problem 3

3. All parts of this question refer to the plot in Figure 1, *which is* $y = f'(x)$ *for some function* $f(x)$. Assume the whole domain of the functions $f(x)$ and $f'(x)$ is the interval $[-3, 3]$ shown.

   (A) (10) Find the critical points of $f(x)$ in the interval shown:
   Answer: _____

   (B) (5) Briefly, in your own words, state how the First Derivative Test distinguishes between local maxima, local minima, and critical points that are neither:

   (C) (5) Identify each of the points you found in part (A) as a local maximum, local minimum, or neither:
   Answer: _____

   (D) (5) Find all the inflection points of $f(x)$.
   Answer: _____

   (E) (5) Over which intervals is $y = f(x)$ concave up? concave down?
   Concave up: _____ Concave down: _____

4. A rectangular poster is to have total area 600 square inches, including blank 1 inch wide margins on all four sides of a central printed area. What overall dimensions will maximize the printed area? The parts of this problem will lead you to the answer.

   (A) (4) Draw a diagram clearly showing the whole poster, the central printed area, and the 1 inch wide blank margins.

   (B) (4) Call the horizontal side of the whole poster x and the vertical side of the whole poster y. Express the total area of poster and the area of the printed region in terms of x and y.

(C) (4) Solve for y in terms of x using the area of the whole poster and substitute into the area of the printed region.

(D) (5) Determine a critical point of the area function.

(E) (4) How do you know your critical point is a maximum of the area?

(F) (4) What are the dimensions x and y of the poster of maximum printed area?