MATH 133 – Calculus with Fundamentals 1
Solutions for Quiz 2 – September 15, 2017

1) (15) In the space below, plot the piece-wise defined function

$$f(x) = \begin{cases} x+3 & \text{if } -1 < x < 1 \\ x^2 - 4x + 2 & \text{if } 1 \le x < 3. \end{cases}$$

Assume the domain is just the interval $(-1, 3)$ indicated.

Solution:

The portion of the graph between $x = -1$ and $x = 1$ is a part of the straight line $y = x + 3$. This has slope $m = 1$ and y-intercept 3, so it goes through the point $(0, 3)$. We *only* get the part of that line starting at $(-1, 2)$ and ending at $(1, 4)$. The endpoints are shown with open circles because $x = -1$ and $x = 1$ are not included in this part of the domain.

At $x = 1$, the formula changes to $f(x) = x^2 - 4x + 2$. Completing the square, we have that half of the coefficient of the x is $-4/2 = -2$ and $(x-2)^2 = x^2 - 4x + 4$. Therefore $x^2 - 4x + 2 = (x-2)^2 - 2$. Therefore, this portion of the graph is a piece of a shifted parabola. The shifts are 2 units to the right and 2 units down. The vertex (lowest point) is at $(2, -2)$. As for the line, we only get the part of the parabola for $1 \le x < 3$. See Figure 1 on the back.

2) (15) Find a formula for the sinusoidal function plotted in Figure 2 on the back.

Solution:

If we think of placing the start of period at $x = 0$, then this looks like a cosine graph. The maximum y-value is 0 and the minimum is -8, so the amplitude is $\frac{0-(-8)}{2} = 4$. The vertical shift is the distance $0 - 4 = -4$. The period of the oscillation is π, so the multiplier B in the general formula $y = A\cos(Bx) + C$ is $B = \frac{2\pi}{\pi} = 2$. So one formula is

$$y = 4\cos(2x) - 4.$$

(There are other correct ones too, but this one is the simplest!)

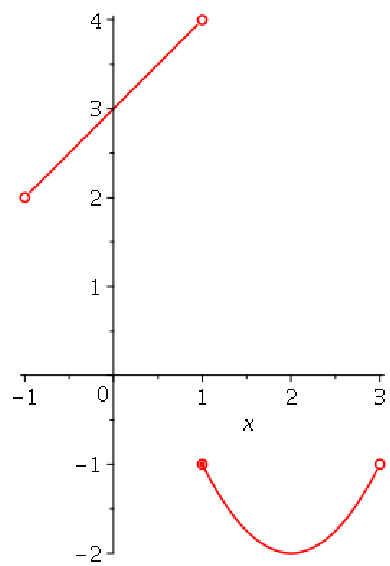

Figure 1: $y = f(x)$ from question 1.

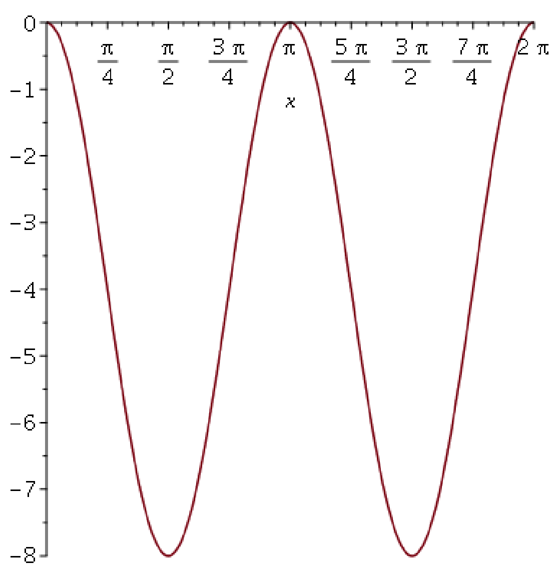

Figure 2: A sinusoidal function