MATH 133 – Calculus with Fundamentals 1
Discussion Day on Inverse Functions
September 14, 2017

Background

If a function from a domain D to a range R has the *one-to-one* property (that is, each y in the range R is $y = f(x)$ for *exactly one* x in the domain D), then we can define a second function f^{-1} with domain R and range D such that

$$x = f^{-1}(y) \text{ exactly when } y = f(x).$$

This new function is called the *inverse function* of f. As we said in the video for today, this f^{-1} is *NOT THE SAME* as $\frac{1}{f}$. We want to work with a few examples of this idea today.

Questions

1) Look at Figure 19 on page 39 of our text.

   (a) Which of the graphs there define functions with the one-to-one property? Why?

   (b) For that ones that do not, could you restrict the domain to get a function satisfying the one-to-one property? (NOTE: This is actually always possible! How would you do it here?)

   (c) Recall from Video 1.5 that there is a geometric way to get the graph of the inverse function from the graph of a one-to-one function. Draw the graphs of the inverse functions for each of the functions from part (a), and then do the same for each restricted domain function from part (b).

(2) For this problem, take it as known that the given functions have the one-to-one property. Find the formula for the inverse function and state what the domain of the inverse is.

   (a) $f(x) = -3x + 5$

   (b) $f(x) = \dfrac{1}{x+1}$.