MATH 133 – Calculus with Fundamentals 1
Piecewise-defined functions
September 11, 2017

Background

In the video for today, we saw a real-world example of the idea of a function defined "piecewise" by different formulas on different sections of its domain. In today's session, you will work with several more examples of these.

Questions

1) For each of the following piecewise-defined functions $f(x)$, sketch the graph and determine the domain and range. (Notes: If values of x are not covered in any of the stated "pieces" or cases, they are *not contained* in the domain. Each "piece" of these examples will be types of functions we have seen before, so you can plot them using ideas we have discussed before. But there is no requirement that the "pieces" link up to make a single connected graph.)

   (a) The function
$$f(x) = \begin{cases} 1 & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

   (b) The function
$$g(x) = \begin{cases} x^2 & \text{if } -3 < x \leq 0 \\ -x^2 & \text{if } 0 < x < 3. \end{cases}$$

   (c) The function
$$h(x) = \begin{cases} x^2 + 2x & \text{if } x < 2 \\ -4x + 9 & \text{if } x \geq 2. \end{cases}$$

2) Each of the following scenarios describes a car trip along a straight road joining two cities 120 miles apart. Mathematically, the road will be the real number line, with your home city at $x = 0$ and the other city is at $x = 120$. For each of the following scenarios, draw a graph of the position of your car as a function of time that describes it, then try to determine piecewise formula(s) for x as a function of t that describe each one. You'll need to use the equation rate $\times$ time $=$ distance for straight line motion at a constant speed.

   Example: You travel at constant speed of 45 miles per hour for one hour then, realizing you are going to be late, you travel the rest of the way at a constant speed of 75 miles per hour for one hour.

   Solution: At time $t = 0$ you are at your home city, so $x(0) = 0$. One hour later you have traveled 45 miles, so $x(1) = 45$. Since you are covering distance at a constant rate, your position satisfies $x(t) = 45t$ for $0 \leq t \leq 1$. Then starting at $t = 1$, you move faster and

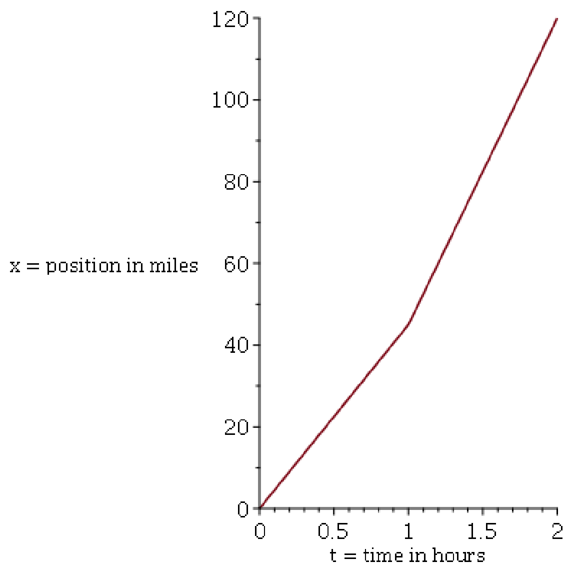

Figure 1: Position vs. Time Graph

$x(2) = 45 + 75 = 120$, so the whole trip takes two hours. During the second hour, your position is changing at 75 miles per hour, which means $x(t)$ is following a linear function $x(t) = 75(t-1) + 45 = 75t - 30$ and the full formula is

$$x(t) = \begin{cases} 45t & \text{if } 0 \leq t \leq 1 \\ 75t - 30 & \text{if } 1 < t \leq 2. \end{cases}$$

This is plotted above in Figure 1. Now you try these:

(a) You travel at a constant speed of 60 miles per hour for one hour, stop for half an hour while a large flock of sheep cross the highway, then continue on at a constant speed of 60 miles per hour until you reached the other city.

(b) You travel for one hour at a constant speed 60 miles per hour, then are engulfed in a dense fog. You turn around and drive home at a constant speed of 30 miles per hour.