MATH 133 – Calculus with Fundamentals 1
Practice on Compositions of Functions
September 8, 2017

Background

Function composition is basically a fancy name for "plugging one function into another function." The slightly tricky thing to remember is that there are two ways to do this with two functions:

1. $f \circ g$ means "plug $g(x)$ into f": $(f \circ g)(x) = f(g(x))$,

2. $g \circ f$ means "plug $f(x)$ into g": $(g \circ f)(x) = g(f(x))$.

Questions

1) For each of the following pairs of functions f, g, compute both compositions $f \circ g$ and $g \circ f$.

   (a) $f(x) = x^2 + 4x + 5$, $g(x) = \sqrt{x}$
   (b) $f(x) = \frac{1}{x^2+1}$, $g(x) = \cos(x)$
   (c) $f(x) = x + \frac{1}{x}$, $g(x) = f(x)$.

2) Express each of the following functions $F(x)$ as $F = f \circ g$ for some functions f, g:

   (a) $F(x) = \sqrt{1 - x^2}$
   (b) $F(x) = 2^{x^2}$
   (c) $F(x) = \frac{1}{x^2+3x+2}$

3) Recall our "rule of thumb" on domains: If $f(x)$ is given by a formula, then unless stated otherwise we take the domain to be the set of all real x that yield a well-defined result in the formula. What are the domains of the functions F in question 2?