MATH 133 – Calculus with Fundamentals 1
Extreme Values
November 17, 2017

Background

Recall from today's video and yesterday's class:

- A continuous function on a closed interval attains a *maximum value* and a *minimum value* on that interval.
- Those maximum and minimum values are attained either at endpoints of the interval or at interior points – nothing deep going on there, just logic(!)
- If the maximum or minimum value is attained at a point $x = c$ *other than an endpoint*, then c must be a *critical point* of f – either a solution of $f'(c) = 0$, or else a place where $f'(c)$ does not exist.
- So to find the maximum and minimum value of a continuous function of a continuous $f(x)$ on a closed interval $[a, b]$, we can:
  (i) Compute $f'(x)$ and find all critical points c in $[a, b]$.
  (ii) Compute $f(a), f(b)$, and $f(c)$ for all critical points found in the first step.
  (iii) Then the maximum value will be the largest of the numbers found in the previous step and the minimum value will be the smallest of those numbers.

Questions

For each of the following functions,

(i) determine all critical points in the given interval,

(ii) compute the values of f at the critical points in the interval, and compute the values at the endpoints of the interval,

(iii) determine the maximum and minimum values of f on the interval.

This is the process sketched above(!)

1. $f(x) = x^3 - 12x^2 + 21x$ on $[0, 11]$.
2. $f(x) = (x^2 + 2x)e^{-x}$ on $[1, 5]$
3. $f(x) = 5\tan^{-1}(x) - x$ on $[-5, 5]$.
4. $f(x) = (t - t^2)^{2/3}$ on $[0, 2]$. (Note: Careful on this one – you should find critical points where $f'(x)$ does not exist.)