MATH 133 – Calculus with Fundamentals 1
The Derivative Function – Graphical Approach
October 24, 2017

Background

Yesterday, we were working with the *derivative* of a function f at x:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists. Since one interpretation of the value $f'(a)$ is

$$f'(a) = \text{slope of tangent line to } y = f(x) \text{ at } x = a,$$

we can start to understand the relation between the graph $y = f(x)$ and the graph $y = f'(x)$ by considering slopes of tangents – in particular:

(a) The intervals where $f'(x) > 0$ (positive slopes of tangent lines to $y = f(x)$)

(b) The intervals where $f'(x) < 0$ (negative slopes of tangent lines to $y = f(x)$)

(c) The points where $f'(x) = 0$ (horizontal tangent lines to $y = f(x)$)

Questions

(1) First look at plots A,B on the back of this page. One is a graph $y = f(x)$ and the other is the graph $y = f'(x)$ for the same function.

   (a) Look at plot A. Over which intervals does that function have tangent lines with positive slope? Over which intervals does that function have tangent lines with negative slope? Where does that graph have horizontal tangent lines (slope 0)? (You'll need to estimate the endpoints from the axis scales.)

   (b) Now look at plot B. Over which intervals does that function have positive values? Over which intervals does that function have negative values? slope? Where does that function equal zero? (You'll need to estimate the endpoints from the axis scales.)

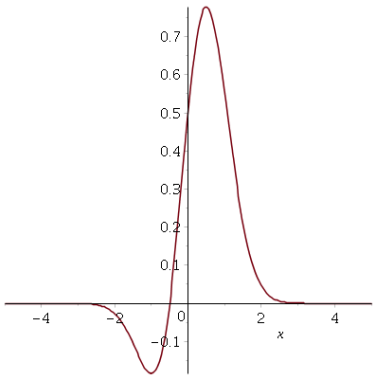
   (c) Which graph is $y = f(x)$ and which is $y = f'(x)$? How can you tell?

(2) Now look at plots C,D. Again one is $y = f(x)$ and one is $y = f'(x)$. Which is which? How can you tell?

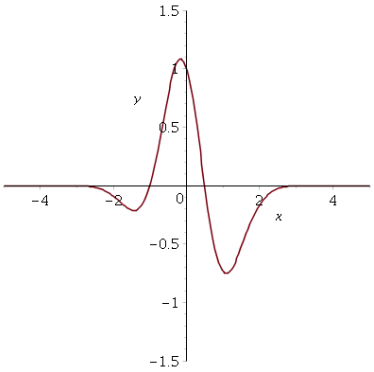
(3) Now refer to plot E. This is $y = f(x)$. Draw a "qualitative" plot of $y = f'(x)$. This means: Don't worry about exact values of $f'(x)$, but generate a plot of $y = f'(x)$ showing the intervals where $f'(x) > 0$, where $f'(x) < 0$ and the points where $f'(x) = 0$.

Figure 1: Plots for Questions 1 and 2

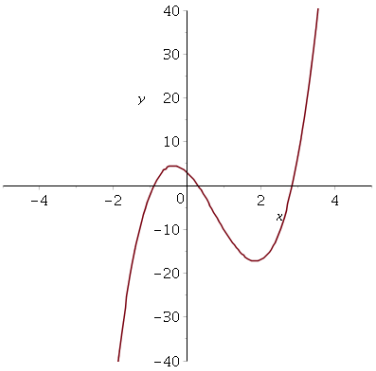
(a) Plot A



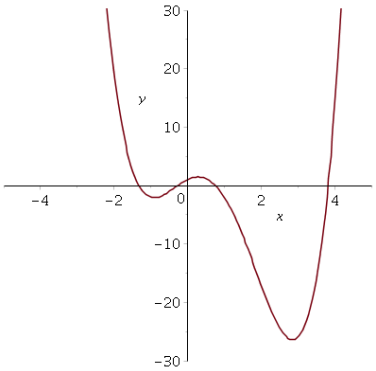
(b) Plot B



(c) Plot C



(d) Plot D



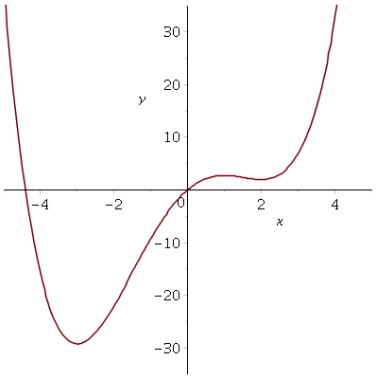


Figure 2: Plot for Question 3