MATH 133 – Calculus with Fundamentals 1
The Derivative Function, part 2
October 23, 2017

Background

On Tuesday, we were working with the *derivative* of a function f at x:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h},$$

provided the limit exists. With today's video we know the following facts about derivatives

(1) If $f(x) = x^n$ for any number n, then $f'(x) = nx^{n-1}$.

(2) If $f'(x)$ exists, then $(kf)'(x)$ exists and $(kf)'(x) = kf'(x)$.

(3) If $f'(x)$ and $g'(x)$ both exist, then so does $(f+g)'(x)$, and $(f+g)'(x) = f'(x) + g'(x)$.

(4) If $f(x) = e^x$, then $f'(x) = e^x$ (that's right, it's the same function(!))

Questions

(1) Compute $f'(x)$ for $f(x) = x^7 + 2e^x$.

(2) Compute $f'(x)$ for $f(x) = x^4 + 3e^x + 7 + 2x^{-1}$.

(3) Make up your own example function of the same type as the one in (2) and compute $f'(x)$ using points (1), (2) and (3) in the Background.

(4) What is $f'(x)$ if $f(x) = e^\pi$? (Be careful!)

The video for today gave the idea for showing point (4) in the Background because for any exponential function $f(x) = b^x$,

$$f'(x) = \lim_{h\to 0} \frac{b^{x+h} - b^x}{h} = b^x \lim_{h\to 0} \frac{b^h - 1}{h}.$$

We said that $b = e$ was the base for the exponential that made

$$\lim_{h\to 0} \frac{b^h - 1}{h} = 1.$$

(5) Take $b = 2.7$ and estimate $\lim_{h\to 0} \frac{(2.7)^h - 1}{h}$ numerically. Then repeat with and $b = 2.8$ and estimate the limit $\lim_{h\to 0} \frac{(2.8)^h - 1}{h}$. What do you conclude about the number e?

(6) Now repeat part (5) with $b = 2.75$. Is e in the interval $[2.7, 2.5]$ or $[2.75, 8]$? Note that you could repeat this process over and over to *zero in* on the decimal expansion of the number e.