

MATH 133 – Calculus with Fundamentals 1  
The Derivative Function  
October 18, 2017

*Background*

Recall that last time, we introduced the *derivative* of a function  $f$  at  $x$ :

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

In today's video we introduced the idea of deriving formulas for  $f'(x)$  for a general  $x$ , and considering  $f'$  as a new function in its own right. We said:

- (1) If  $f(x) = x^n$  for any number  $n$ , then  $f'(x) = nx^{n-1}$ .
- (2) If  $f'(x)$  exists, then so does  $(kf)'(x)$  and  $(kf)'(x) = kf'(x)$ .
- (3) If  $f'(x)$  and  $g'(x)$  both exist, then so does  $(f+g)'(x)$ , and  $(f+g)'(x) = f'(x) + g'(x)$ .

*Questions*

- (1) Compute  $f'(x)$  for  $f(x) = x^{5/3}$  using point (1) in the Background above.
- (2) Compute  $f'(x)$  for  $f(x) = x^4 + 5x^2 + 7 + 2x^{-3}$  using points (1), (2), and (3) in the Background above.
- (3) Make up your own example function of the same type as the one in (2) and compute  $f'(x)$  using points (1), (2), and (3) in the Background.
- (4) What is  $f'(x)$  if  $f(x) = \pi^\pi$ ? (Be careful!)

The video for today gave the idea for showing point (1) in the Background when  $n$  is a positive integer, and stated that the same formula works for all  $n$ . Let's verify this in some special cases.

- (5) Verify the formula in point (1) for  $f(x) = x^4$ . That is, you want to show  $f'(x) = 4x^3$  using the definition of  $f'(x)$ .
- (6) Verify the formula in point (1) for  $f(x) = \frac{1}{x^2} = x^{-2}$ . That is, you want to show  $f'(x) = -2x^{-3}$  using the definition of  $f'(x)$ .
- (7) Why does point (2) in the background work? Explain in terms of the limit definition, and in terms of how the graphs  $y = f(x)$  and  $y = kf(x)$  are related.

Finally, it is important to realize that the limit in the definition of  $f'(x)$  can fail to exist at some of the  $x$  in the domain of  $f$ .

(9) Consider  $y = |x|$  (the absolute value function). Does

$$f'(0) = \lim_{h \rightarrow 0} \frac{|h| - 0}{h}$$

exist? Why or why not? Does this make sense in terms of the graph  $y = |x|$ ?