MATH 133 – Calculus with Fundamentals 1
Practice on Absolute Values and Intervals
August 31, 2017

Background

Recall from the video "lecture" for today that the absolute value of a real number is defined as

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

Also, we have several different types of *intervals* in the real number system written as follows:

$$(a, b) = \{x \mid a < x < b\} \tag{1}$$
$$[a, b] = \{x \mid a \leq x \leq b\} \tag{2}$$
$$[a, b) = \{x \mid a \leq x < b\} \tag{3}$$
$$(a, b] = \{x \mid a < x \leq b\}. \tag{4}$$

Today we will practice on some problems related to these ideas.

Questions

0) (Warm-up) Real numbers can all be expressed by (possibly infinite or non-repeating) decimal expansions. But exactly what does a decimal expansion like

$$\pi = 3.1415926\cdots$$

mean? What does a finite decimal expansion like $\frac{15}{8} = 1.875$ mean? (Hint: It's really a way of writing the number as a sum of fractions with denominators of a particular form.)

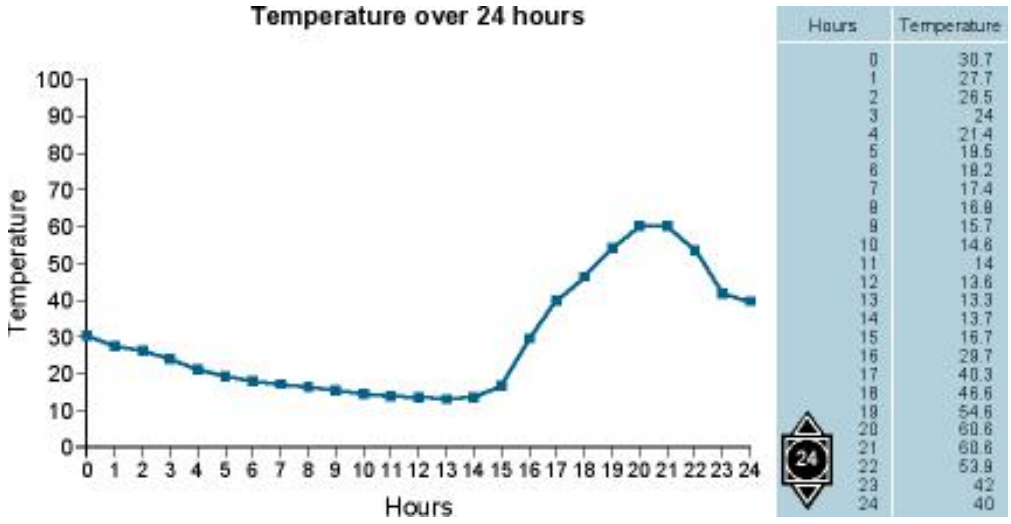
1) The set of real numbers x satisfying $|x| < 17$ is an interval. Write it using the appropriate interval notation.

2) The set of real numbers x satisfying $|x - 3| \leq 1$ is also an interval. Write it using the appropriate interval notation.

3) The set of real numbers x satisfying $|3x + 1| < 4$ is also an interval. Write it using the appropriate interval notation. (Hint: One way is to factor out the 3, divide through, then proceed as in 2). Another is to proceed algebraically from the equivalent inequalities $-4 < 3x + 1 < 4$.)

4) Write the interval $(5, 9)$ in the form: "the set of all x such that $|x - c| < r$" for some real numbers c and r.

5) Same as question 4, but for the interval $\left[-\frac{2}{5}, 3\right]$.



Hours	Temperature
0	30.7
1	27.7
2	26.5
3	24
4	21.4
5	19.5
6	18.2
7	17.4
8	16.8
9	15.7
10	14.6
11	14
12	13.6
13	13.3
14	13.7
15	16.7
16	28.7
17	40.3
18	46.6
19	54.6
20	60.6
21	60.6
22	53.9
23	42
24	40

Figure 1: Figure for Question 6

6) Using the graph of temperature versus time over a 24-hour period at a particular location above, answer these questions:

   (a) Write the collection of all *temperatures* measured over the 24 hour period as an interval $[a, b]$ for suitable temperature values a, b in degrees Fahrenheit. (There's an unstated assumption behind this question. Can you see what that is? Does it seem reasonable to make that assumption?)

   (b) Estimate the interval $[c, d]$ of *times* in hours for which the temperature was 16.8 degrees or less.

   (c) At how many different times does it appear that the temperature was exactly 40 degrees F?

   Note: The temperature values at the start of each hour are given in the table at the right.