MATH 133 – Calculus with Fundamentals 1
Solution for 4.2/74 from Problem Set 8 – Part B
December 5, 2015

- The formula for the concentration is given as

$$C(t) = \frac{.016t}{t^2 + 4t + 4}.$$

- But the problem becomes much simpler if you rewrite this as

$$C(t) = (.016) \cdot \frac{t}{(t+2)^2}$$

- Note: I pulled the constant factor .016 out and factored the denominator noticing that it is a perfect square.
- We want to find the maximum and minimum values of this function on the interval $[0, 8]$.
- First we differentiate $C(t)$ using the quotient rule, simplify, and then set the derivative equal to find the critical points:

$$C'(t) = (.016) \cdot \frac{(t+2)^2 \cdot 1 - t \cdot 2(t+2)}{(t+2)^4} = (.016) \cdot \frac{(t+2)(2-t)}{(t+2)^4} = (.016) \cdot \frac{2-t}{(t+2)^3},$$

so $C'(t) = 0$ when $t = 2$.

- Now we compute:

$$\begin{aligned} C(0) &= 0 \\ C(2) &= \frac{.032}{16} = .002 \\ C(8) &= \frac{.128}{100} = .00128. \end{aligned}$$

- So the maximum value is $C(2) = .002$ and the minimum value is $C(0) = 0$.

Some comments:

(a) A lot of people got very tied up in dealing with the "complicated" decimal number coefficients that you get if you don't start out by pulling out the constant factor .016. *Don't forget the constant multiple rule for derivatives:*

$$\frac{d}{dx}(kf(x)) = kf'(x).$$

In words, *the derivative of a constant k times a function is the same constant k times the derivative of the function.* You can (and usually should) do something like the above whenever you can pull a constant factor out of a whole function you need to differentiate.

(b) If you don't factor the denominator, you will get something like this when you apply the quotient rule:

$$C'(t) = (.016) \cdot \frac{(t^2 + 4t + 4) \cdot 1 - t(2t + 4)}{(t^2 + 4t + 4)^2} = (.016) \frac{-t^2 + 4}{(t^2 + 4t + 4)^2}$$

This makes it look as though both solutions of $-t^2 + 4 = 0$, namely, $t = \pm 2$ are critical points. *However*, the root $t = -2$ *is not a critical point* because it is not in the domain of the original $C(t)$ (the function $C(t)$ has an infinite discontinuity – a vertical asymptote – there). Check the definition of critical points on page 202 of the text book – critical points of a function are points where the function is defined, but where either the derivative is zero, or the derivative is undefined. And in any case, $t = -2$ is not in the interval $[0, 8]$ we are interested in, so that should be ignored.